# A COMPENDIOUS SYSTEM OF Natural Philoſophy,

With NOTES
Containing the MATHEMATICAL DEMONSTRATIONS, and ſome occaſional REMARKS.

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## PART IV. Continued.

CONTAINING THE
DESCRIPTION
OF THE
ORRERY *and* GLOBES,
The PRINCIPLES of
CHRONOLOGY;
Together with the Phyſical Cauſe of the Motion of the
HEAVENLY BODIES.

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# CHAP. XV.

## *The* Deſcription *and* Uſe *of the* Orrery *and the* Globes.

I. THE Orrery is a Machine deſigned to repreſent at one View the real Motions of the Heavenly Bodies. In order to this, in the middle of a large Circle deſigned to repreſent the Ecliptic, is fixed a Globe repreſenting the Sun. Next to the Sun a ſmall Ball repreſenting *Mercury.* Next to this is *Venus* repreſented by a larger. At a greater Diſtance ſtill from the Sun, is the Earth repreſented by an Ivory Ball. Round the Earth, at ſome Diſtance from it, is a Ring to expreſs the Orbit of the Moon, making an Angle with the Circle that repreſents the Ecliptic, and thereby ſhewing the Inclination they have to each other in the Heavens, and alſo the Line of the Nodes. Within this Ring is a ſmall Ivory Ball repreſenting the Moon, and having a black Cap or Caſe, which always covers that Hemiſphere which is turned from the Sun; and thereby diſtinguiſheth the enlightned Part from the dark one. Beyond theſe is ſeen *Mars*, then *Jupiter* and its four Moons, and outermoſt of all is *Saturn* with its Ring and five Moons.

All theſe are fixed upou ſmall Stems which ſeverally repreſent their Axes, each of which has its peculiar and proper Inclination to the Plane of that Circle which we ſaid was to repreſent the Ecliptic: And when the Machine is put in Motion, all theſe Bodies move round that which repreſents the Sun, and at the ſame time both that and all thoſe which repreſent ſuch of the Planets as have been obſerved to have a Rotation about their Axes, turn round upon the ſaid Stems, and in their proper Times. The Moons alſo revolve about their Primaries at the ſame Time, and the Ring that repreſents the Orbit of the Moon has likewiſe its proper Motion, whereby that of its Nodes is alſo expreſſed.

Theſe Bodies are all kept in Motion by Wheelwork conceal'd in a Caſe underneath, and turned by a ſmall Winch in the ſide of the Caſe; one Turn anſwering to a Revolution of the Ball which repreſents the Earth, about its Axis; and conſequently to 24 Hours of Time. So that 27 Turns cauſes the Sun to turn once about its Axis, 365 carries the Earth about the Sun, 27 the Moon about the Earth, &c. So that not only the Motion of the Heavenly Bodies may be repreſented, but their Situations with reſpect to each other may be ſhewn for any Time either paſt or to come; (*a*) by

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(*a*) This and the next Paragraph muſt be underſtood with ſome Limitation, Machines of this kind being ſufficiently adapted to common Uſe, though not accurate enough for this purpoſe; none of the Machines of this kind which have yet been made, being ſo contrived as to ſhew this with Accuracy.

by giving the Winch a proper Number of Turns backwards or forwards as the Caſe requires.

There is uſually alſo a Lamp in a Caſe, with one or more convex Glaſſes in the ſide of it, to throw a ſtrong Light directly forwards; this Lamp being put in the Place of the Sun, and that ſide of the Caſe which has the Glaſs in it directed towards the Earth, it being alſo contrived to turn according to the Motion of the Earth, a ſtrong Light is thereby continually thrown upon it, and the Moon, in whatever part of its Orbit it is, and ſo not only the Times in which the Eclipſes of the Sun and Moon, that is, properly of the Earth and Moon will happen, are ſhewn, but they themſelves are duly repreſented. (*a*)

II. As

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(*a*) Artificers generally erect upon the Ecliptic ſome Semicircles, to repreſent ſome of the principal Circles of the Heavens. But this is wrong, and tends to Confuſion; becauſe theſe Circles being only imaginary, and ariſing from the *apparent* Motions of the Heavenly Bodies, ought to have no Place in the Orrery: Which as it expreſſes their *real* Motions, ſerves to *explain*, yet is by no means intended to *repreſent* their *Phænomena*; the Phænomena and real Motions being in ſeveral Caſes directly contrary to one another. To be particular, they have in ſome affixed the two Extremities of a Semicircle, which they call a moveable Horizon by two Hinges to the Ecliptic, at the firſt Degree of *Aries* and *Libra*; this is as if all Horizons neceſſarily paſſed through theſe Points, which is impoſſible, becauſe there are as many Horizons as Points upon the Earth. So that the putting on this Circle is not only an Impropriety common to the reſt, but is attended with a neceſſary Blunder. A learned Expoſitor of the Orrery ſays, ' when this Machine is ſet to any Latitude, ' (for they have a Contrivance for that too) ſet the moveable ' Horizon to the ſame Degree upon the Meridian, and you may

' form

II. As the Orrery is a Machine proper to repreſent the real Motions of the Heavenly Bodies, ſo the Globes are adapted to their apparent Motions: And that I may give the Reader a juſt and adequate *Idea* of them, I ſhall premiſe the following Deſcription of a Machine, which though imaginary, will greatly tend to the true Conception both of their Nature and Uſes.

In Order to this, let us conceive a large hollow Sphere repreſenting the inſide of the Heavens, and on the inner Surface of this Sphere let the Sun, Moon and Stars, be ſuppoſed to be delineated in their proper Places; and let the Sphere be put into a Frame, and hang there in ſuch Manner, as that it may be capable of turning about an Axis: And let it be ſo contrived, that either end of the Axis may be raiſed or lowered at Pleaſure.

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‘ form an Idea of the reſpective Altitudes, or Depreſſions of the
‘ Planets above or below the Horizon, according to their reſpec-
‘ tive Poſitions, with regard to the Meridian.’

Now the Idea which the Motion of the Orrery in this Poſition gives me, is that the whole Earth is one half of the Year above the Horizon, and the other half below it. For the Ball which repreſents it in the Orrery is manifeſtly ſo, while it performs its Revolution about the Sun.

Again, ſome Artiſts, in order to make the Orrery repreſent the ſeveral Phaſes of the Moon, make one half of the Ball that repreſents her black; in which caſe they are obliged to contrive it ſo, that the ſame ſide of the Moon ſhall not be always turned towards the Earth; which is contrary to her real Motion, or that the dark ſide ſhall ſometimes be turned towards the Sun, which is abſurd.

In the midſt of this Sphere, let there be a ſmall Ball to repreſent the Earth, but ſo fixed there, as not to turn round with the Sphere, and let a Spectator be ſuppoſed to be placed on the uppermoſt Point of this Ball; and to prevent the lower half of the Sphere from being viſible to him, let a Plane, that is, a thin broad Subſtance, a Plate of Braſs for Inſtance, be ſuppoſed fixed on that Point of the Ball where the Spectator ſtands, and to reach on all ſides as far as the inner Surface of the Sphere: This will repreſent the Horizon of that Spectator; becauſe it ſeparates the upper half of the Sphere, which we are to ſuppoſe viſible to this Spectator, from the lower, which we muſt ſuppoſe to be out of his Sight. Then, as is moſt obvious, by turning this Sphere once round, while the Ball, the Spectator and the aforeſaid Plane remain unmoved, the riſing and ſetting of the Sun, Moon and Stars, by their paſſing by the Edges of this Plane, will be duly repreſented: (*a*) And thoſe Stars which do not go down ſo low as the Plane (as is evident if one End of the Axis of the Sphere be above it, ſome will not,) will be thoſe which never ſet to a Spectator on that Part of the Earth, which correſponds with the

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(*a*) The Reader is to ſuppoſe here, that in this caſe the Sphere is not hung in its Frame in ſuch manner, that one Extremity of its Axis ſhall be directly over the Spectator; becauſe in that caſe none of the Heavenly Bodies in turning the Sphere, will paſs by the Edge of the Plane.

uppermoſt Point of the Ball; thoſe which during the whole Revolution, do not riſe ſo high as to come above it, will be thoſe which never riſe.

Again, if we ſuppoſe a Circle as large as the inſide of the Sphere will admit of, to be placed within it, in ſuch manner as to have the Axis of the Sphere paſſing through two oppoſite Points of it, and alſo to have one Point of it directly over the Head of the Spectator, and the oppoſite one directly under his Feet, and to remain unmoved while the Sphere turns; This will repreſent the Meridian of that Spectator; and ſo the Culminations of the Heavenly Bodies, that is, their paſſing the Meridian in the Heavens, will be exhibited by their paſſing by this Circle.

Farther, if we alſo imagine an Index like the Hand of a Clock, to be fixed upon one of the Extremities of the Axis of the Sphere, in ſuch manner as to turn with the Sphere, and to point continually to a Circle fixed on the Frame, with the 24 Hours delineated upon it; and if this Index be ſet to point to the Hour 12, when the Sun's Place upon the Sphere for any given Day, is at the Circle that repreſents the Meridian, then by turning the Sphere this way or that, till the Place of the Sun or any of the Stars comes to the Plane abovementioned, the Index moving along with the Sphere, will point to the Hour at which they riſe or ſet that Day: And if we obſerve to what

Hour the Index points, when any of them croſſes the Meridian within the Sphere, the Time of their Culmination, or Southing, will be expreſſed: And ſo by this Machine all Problems relating to the riſing, ſetting and ſouthing of the Sun, Moon and Stars, may be ſolved. The Reaſon why the Index muſt be ſet to point to the Hour 12, when the Place of the Sun is brought to the Meridian, is that it may point to that Hour, which the real Sun indicates, when the Heavens ſtand in a Poſition correſponding with that of our imaginary Sphere.

But ſince by means of the annual Motion of the Earth round the Sun, the Sun continually ſhifts its apparent Place in the Heavens in ſuch manner, as to ſeem to deſcribe a great Circle once a Year; it is evident that its Place cannot be delineated on the inſide of this Sphere as we have ſuppoſed, unleſs it be for a Day or a few Hours, during which Time its change of Place is ſo ſmall that it may be neglected. In like manner the Planets are continually changing their apparent Places in the Heavens, by means of their Motions in their reſpective Orbits; neither therefore can their Places be aſſigned them, unleſs for a very ſmall interval of Time. Let us then imagine both the Sun and the Planets to be repreſented by little Balls adhering to the inner Surface of the ſaid Sphere; but let it be ſo contrived, that each Ball ſhall ſhift its Place with every Turn of the Sphere, juſt ſo much, and in the ſame Direction that the Sun or other heavenly Body the Ball repreſents, appears to ſhift its Place in the Heavens in one Day: And let the Index ſhift with the Sun, ſo that it may always point to 12, when the Place of the Sun comes to the Meridian.

Then, as is moſt obvious, will our Machine be ſo adapted, that upon turning it round continually, the Change of the Time of the riſing and ſetting of the Sun, its different Meridian Altitude, the viciſſitude and length of Day and Night, the alteration of the Time of the riſing and ſetting of the fixed Stars and of the Planets, for ſo many Days running, as is the Number of Turns given to the Globe, will all be exhibited to View: And conſequently by turning the Sphere 365 Times round, the Succeſſion of the Seaſons will be repreſented throughout the Year with abundance more, or rather will all the remaining Phænomena of the Heavens.

This Machine, as hitherto deſcribed, ſerves only to exhibit the Phænomena of the Heavens as they appear on one Point of the Earth's Surface, *viz.* that which is repreſented by the uppermoſt Point of the Ball, or that where the Spectator was ſuppoſed to ſtand. But if we imagine the Plane which was put as a Boundary to the Spectator's Sight, to be applied to ſome other Point of the Ball and fixed there, then the Motion of the Sphere will exhibit the Phænomena, as they would appear to a

Spectator on that Part of the Earth, and ſo for the reſt; or which comes to the ſame thing, if while the Plane remains unmoved, one of the Poles of the Sphere be elevated above it as many Degrees as the correſponding Pole in the Heavens is above the Horizon of a Spectator, on any Part of the Earth; then will the Revolution of the Sphere repreſent the Phænomena as they appear to that Spectator. For it is the ſame Thing whether the Pole be lifted up from the Horizon, or the Horizon put down below the Pole. (*a*)

Now this Machine (except in ſome few Particulars, in which it vaſtly excels the celeſtial Globe) is exactly the ſame with it as to its Uſes. Thus, the Globe turns upon an Axis, either End of which may be raiſed or depreſſed at Pleaſure; and on one of them is an Index that turns with the Globe, and points to a fixed Circle that has the Hours upon it. There is no Ball in the middle, as we ſuppoſed in the Machine, but the Place of the Spectator is al-

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(*a*) By means of an Orrery to repreſent the real Motions of the heavenly Bodies, and ſuch a Machine as above deſcribed, to ſhew afterwards how the apparent Motions of thoſe Bodies depend on, and are connected with their real ones, the Elements of Aſtronomy might be taught much better than by any thing that has as yet been contrived; or rather, if I may ſo ſay, the Science itſelf might thereby be exhibited to View. For the Orrery, though it admirably well illuſtrates ſome Things, particularly the Eclipſes, and in general what relates to the Motions of the Planets about the Sun, yet it ſhews the Appearances or Phænomena ariſing from thence to a Spectator, ſuppoſed to be at reſt upon the Surface of the Earth, but in a forced and unnatural Way.

ways ſuppoſed to be there; and inſtead of the Plane we ſuppoſed to touch that Part of the Ball on which the Spectator ſtands, and to be extended every way as far as the inner Surface of the Sphere, there is a broad wooden Circle on the outſide of the Globe, (which is Part of the Frame of it) exactly where that Plane would paſs, was it continued beyond the Surface of the Globe, and ſo anſwers the Uſes of that Plane. In the next Place, inſtead of the Circle we ſuppoſed fixed within the Sphere and paſſing through the *Zenith* and *Nadir* of the Spectator, and through which the Axis of the Sphere paſſes, there is a brazen Circle on the outſide of the Globe, and in which it ſwings. And inſtead of the Stars being on the inner Surface, they are here on the outer one, and the Conſtellations are delineated in their proper Forms. Then as to the Places of the Sun and Planets, Artificers are content to leave them quite out, and only draw a great Circle repreſenting the Path of the Sun in its annual Courſe through the Heavens, *viz.* the Ecliptic, and diſtinguiſh it into 12 Parts by the Signs thereof, and each Sign into 30 leſs, which are the Degrees thereof. When therefore a Problem is to be determined by the Globe, requiring that the Sun ſhould be delineated in its proper Place, you are to look out the Place of the Sun for the given Time in ſome aſtronomical Tables, Ephemeris or Almanack for that Purpoſe, or on the broad Surface of the

Horizon of the Globe, where there is uſually a Table ſhewing the Sun's Place in the Ecliptic for every Day of the Year, and then to mark its Place on the Line which we ſaid repreſented the Path of the Sun upon the Globe. In like manner, if a Problem is to be ſolved requiring the Place of any of the Planets, the Place of the Planet for the Time given muſt be looked out in ſome Ephemeris or Almanack, of which there are many publiſh'd yearly by *Parker*, and others, for that Purpoſe, containing Tables expreſſing the Longitude and Latitude of each Planet for every Day of the Year; and marking the Place of the Planet upon the Globe, by ſticking on a Patch, or the like, and ſetting the Index according to the Place of the Sun for the given Time, and turning the Globe about, the Index will ſhew the Time of that Planet's coming to either Side of the Horizon, to the Meridian, or to any Height above or below the Horizon, *&c.*

If then the Reader will form to himſelf a true Idea of the Machine I have deſcribed, and of the manner in which ſuch a Machine is capable of repreſenting the apparent Motions of the heavenly Bodies, he cannot fail of having a thorough Notion of the Nature of the celeſtial Globe; the main Difference between them being only this, *viz.* that what we have ſuppoſed in the one to be on the Inſide, is either actually on the other on the Outſide, or to be fixed on, or to have its Place marked there, as Occaſion requires. The uſes both of the celeſtial and terreſtrial Globe, as to particular Caſes, will be further illuſtrated by the Inſtances at the End of this Chapter, after we have conſidered ſome other Particulars relating to each.

The Circles delineated on the celeſtial Globe are uſually, beſides the Ecliptic abovemention'd, the Equinoctial Circle at an equal Diſtance from either Pole, and two Parallels of it, *viz.* one on either Side, to repreſent the apparent diurnal Motion of the Sun, when at its greateſt Diſtance from the Equator, called the Tropics; and at the like Diſtance from either Pole, two more Parallels call'd the polar Circles. Both the Ecliptic and the Equinoctial are divided into Degrees, beginning at the Point that repreſents the vernal Equinox, which is one of thoſe where they croſs each other; the Equinoctial into 360, but the Ecliptic into 12 Signs, diſtinguiſhed by their proper Characters, every one of which is ſubdivided into 30 Degrees (*a*). There are alſo ſix Secondaries of the Equator, and as many

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(*a*) As to the Diviſions of the Ecliptic, it is to be obſerved, that the Degrees belonging to each Sign are not to be found where the Conſtellation whoſe Name it bears is delineated, but where the Character peculiar to that Sign is placed. The Reaſon of which is, becauſe the firſt Degree of *Aries*, from whence all the reſt are counted, being where the Equinoctial cuts the Equator, is not in this Age in any Part of the Conſtellation that bears that Name, as was accounted for above.

of the Ecliptic cutting it in the firſt Degree of each Sign; two of which are alſo Secondaries of the Equator, *viz.* the Equinoctial and Solſtitial Colures. Further, the Globe is ſo hung in its Frame, that that Side only of the brazen Meridian which has the Degrees marked upon it, paſſes through the Axis of it, which Side only, in all Caſes where that Meridian is conſidered, is therefore ſuppoſed to repreſent it. Upon the broad Surface of the wooden Horizon, beſides the innermoſt Circle divided into its proper Degrees, there is a Table ſhewing what Degree of the Ecliptic the Sun is in for every Day of the Year, as was obſerved above; in order to which the 12 Signs of the Ecliptic, with the Degrees of each, and the 12 Calendar Months, are ſo placed over-againſt one another, both according to the *Julian* and *Gregorian* Account, that is, according to the old and new Stile, as that the Days of the one may duly anſwer to the correſponding Places of the Sun upon the other. There is alſo a Circle, repreſenting the Winds or Points of the Compaſs.

There is alſo belonging to the Globe a *Quadrant of Altitude.* It is a long and narrow Plate of Braſs, made ſo thin and plyant, that the whole Length of it may be applied cloſe to the Surface of the Globe, and having the Degrees upon it, (*a*) ſerves to meaſure the Diſtan-

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(*a*) That Edge, which has the Degrees neareſt it, is called its *graduated Edge,* and is the only Part of it that is conſidered in the uſing it.

ces of Places on the Globes; or being fixed on to the uppermoſt Point of the Meridian of the celeſtial Globe, (for which Purpoſe it has a Screw at one End) and having the other End thruſt in between the Horizon and the Globe, meaſures the Height of a Star, or other Heavenly Body above the Horizon, *&c.* From which latter Uſe it has its Name.

The terreſtrial Globe differs from the celeſtial principally, in that inſtead of the Conſtellations, the ſeveral Countries of the Earth are delineated thereon; and the Circles on this Globe are uſually the Ecliptic, and the Equator, on which are numbered the Degrees of Longitude, (*a*) the two Tropics, the two polar Circles, Secondaries of the Equator cutting it at every tenth Degree, and Parallels alſo of it at every tenth Degree on each Side. There are alſo drawn upon it Rhumb Lines, or thoſe which a Ship, keeping the ſame Courſe, that is, ſteering continually in the ſame Point of the Compaſs, deſcribes upon the Surface of the Sea. The Property of a Line of this kind, is that it cuts all the Meridians it paſſes through, under the ſame Angle, and therefore is a kind of Spiral continually approaching towards ſome Point. This is called a *Loxodromic* Curve.

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(*a*) On ſome Globes, the Degrees of Longitude are number'd both ways from the firſt Meridian, till you come to the oppoſite Point, and the Longitude is accordingly diſtinguiſhed into Eaſt and Weſt.

The Lines of Variation deſcribed upon the terreſtrial Globe, ſhew the Direction of the magnetic Needle, in thoſe Places in which it does not point directly towards the North. For it is obſerved, that the Needle in ſome Parts of the World, declines from the North to the Eaſt, and in others to the Weſt.

Further, on the Pedeſtal of ſome Globes there is a Mariner's Compaſs fixed, by the help of which, the brazen Meridian of the Globe may be ſet North and South. (*a*)

After this Deſcription of the Globes, and general Idea of the Uſe thereof, it may now be proper to exemplify the latter by the following Inſtances.

And firſt of the celeſtial Globe.

### Problem I. *To rectify the* Globe *to the Latitude of any Place upon the Surface of the Earth.*

If the Place be in the Northern Hemiſphere of the Earth, ſlide the Meridian within the Notches of the Horizon, till there be as many

(*a*) The Armillary Sphere comes the neareſt of any we have to that imaginary Machine deſcribed above, for it is hollow, and has a Ball in the middle to repreſent the Earth; but then in all other Reſpects it is like a Globe, except that the Surface is cut away every where, (unleſs where a Circle is deſcribed) that you may ſee the Ball in the middle. There is one Circle left broad enough to repreſent the Zodiac, on which the Places of the Sun and Planets may be marked, as Occaſion requires.

Degrees of the Meridian between the North Pole and the Horizon, as are equal to the Latitude of the Place (*a*). If the Place be in the Southern Hemiſphere, the Southern Pole muſt be raiſed to a like Height above the Horizon.

## PROB. II. *To find what* Stars *never riſe or never ſet in any* Place *or* Latitude *given.*

The Globe being rectified to the given Latitude of the Place by Problem the firſt, thoſe Stars which paſs not under the wooden Horizon of the Globe, during its whole Revolution, never ſet to a Spectator in that Latitude: And ſuch as do not come up above it, never aſcend above that of the Spectator, and ſo never are ſeen to riſe.

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(*a*) The Reaſon of this is, becauſe to a Spectator, upon any Part of the Earth, the Elevation of the Pole is equal to the Latitude of the Place; and the Globe is ſaid to be *rectified* by this Operation, becauſe, by this means, that Point of the little Ball, (for ſuch we muſt always ſuppoſe to be in the middle of the Globe, to repreſent the Earth) that correſponds to the Place of the Spectator upon the Earth, is brought to the uppermoſt Point thereof; and ſo the Horizon thereof becomes coincident with the Plane of the wooden Horizon that ſurrounds the Globe, and the Poles of the Globe having the like Situation with reſpect to the uppermoſt Point of the Ball, that the Poles of the World have with reſpect to the Place of the Spectator; it is evident that the Revolution of the Globe is fitly diſpoſed to exhibit that of the Heavens, as it would appear to that Spectator.

## PROB. III. *To find the* Sun's *Place in the Ecliptic, for any Day given.*

If the Day given be according to the *Julian* Account, or old Stile, then in the wooden Horizon of the Globe, in the *Julian* Kalendar, otherwiſe in the *Gregorian*, find the Day given, over-againſt which in a Circle, which contains the ſeveral Signs and Degrees of the Ecliptic, is that Degree in which the Sun is in that Day. Though it may be found more accurately by an Ephemeris calculated for that Year. Then in the Ecliptic, delineated on the Surface of the Globe, look for the ſame Degree, and that is the Place of the Sun upon the Globe for that Day.

## PROB. IV. *The Latitude of a Place being given, to find the Time of the* Sun's *riſing and ſetting, and conſequently the Length of Day and Night in that Place.*

Having rectified the Globe according to the Latitude, (by *Prob.* 1.) and found the Sun's Place in the Ecliptic (by *Prob.* 3.) bring the latter to the graduated Edge of the brazen Meridian, and holding the Globe in that Poſition, ſet the Hour Index to 12 at Noon; (*a*) then turn the Globe till the Sun's Place in the Ecliptic comes to the eaſtern Part of the Horizon, and the Index will point to the Hour at which the Sun riſes; and if you turn the Globe till the Sun's Place cuts the weſtern Side of the Horizon, the Index will ſhew the Time of its ſetting.

## PROB. V. *To find the Hour of the Day, by the Height of the* Sun.

Firſt, by the Help of a Quadrant or ſome other Inſtrument for that Purpoſe, obſerve how many Degrees the Sun is elevated above the Horizon. Then the Globe being rectified to the Latitude of the Place, and the Sun's Place in the Ecliptic found, and the Hour Index duly placed, that is, ſo as to point to 12 at Noon, when the Sun's Place is under the Meridian; and ſcrewing the Quadrant of Altitude on to the Meridian at the Zenith, (*i. e.*) at ſo many Degrees from the Equator, as is the Latitude of the Place. Turn the Quadrant of Altitude and the Globe backwards and for-

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(*a*) Becauſe when the Place of the Sun is in the Meridian, the Globe repreſents the Poſition of the Heavens, in reſpect of the given Place, as they are at Noon, or 12 o' Clock of the Day given. And conſequently, if the Hour Index be put to 12, when the Globe is in this Poſition, as it turns with the Globe, it will point to the Hour that correſponds with any other Poſition of the Globe that Day.

wards, till the Sun's Place in the Ecliptic lies under that Degree on the graduated Edge of the Quadrant of Altitude, which anſwers to the Sun's Height ſound as above, then will the Hour Index point out the Time of the Day. (*a*)

## PROB. VI. *To find the Hour of the Night, by the Height of a* Star.

The Height of the Star being ſound by an Inſtrument, the Globe rectified, and the Quadrant of Altitude fixed, as in the foregoing Problem, move the Globe and the Quadrant, till the Star comes under its graduated Edge, at that Degree which expreſſes the Height of the Star above the Horizon; then will the Hour Index point to the Hour of the Night.

## PROB. VII. *To find the Place of a* Star *or* Planet *upon the* Globe, *its Longitude and Latitude being given.*

Lay the firſt Degree of the Quadrant of Altitude, upon that Degree of the Ecliptic, which expreſſes the given Longitude, and the ninetieth Degree thereof on the Pole of the

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(*a*) Becauſe when the Sun's Place is brought to this Point of the Globe, it is ſo many Degrees above the Horizon, as the Sun itſelf is above that of the Spectator.

Ecliptic, (that is, where its Secondaries croſs each other upon the Globe) either Northern or Southern, as the given Latitude is North or South: Then look upon the Quadrant for the Degree of Latitude given, and the Point of the Globe which is under that, is the Place of the Star or Planet ſought. (*a*)

## PROB. VIII. *To repreſent the Appearance of the* Heavens *at any Time of the Night, that is, to ſhew the Situation of the fixed* Stars *at that Time.*

Rectify the Globe, and ſet the Hour Index to the Sun's Place in the Ecliptic, and turn the Globe till the Index points to the Hour of the Night. (*b*)

Prob.

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(*a*) The Reaſon of this is, becauſe the Quadrant of Altitude in this Situation, repreſents a Secondary of the Ecliptic, an Arch of which interſected between the Star and the Ecliptic is its Latitude; and an Arch of the Ecliptic between the Point where it is interſected by a Secondary paſſing through the Star, and the firſt Degree of *Aries*, is its Longitude.

(*b*) For then the Situation of the Stars upon the Globe, with reſpect to the Horizon and Meridian, will be ſimilar to that of the Stars in the Heavens; *viz.* thoſe which appear on the Eaſtern Side the Globe in the Horizon, will then be riſing, and thoſe in the Horizon at the other Side will be ſetting, thoſe which are under the Meridian, will then be culminating, &c.

And if by means of a Compaſs upon the Pedeſtal of the Globe, you ſet the brazen Meridian North and South, ſo that the South Part

## PROB. IX. *The Day of the Year and the Latitude of the Place being given, to find the beginning of the Morning, and End of the Evening* Twilight.

The Globe being rectified to the Latitude of the Place, and the Hour Index ſet to the Place of the Sun in the Ecliptic, elevate that Degree of the Ecliptic which is diametrically oppoſite to the Sun's Place, 18 Degrees above the Weſt Side of the Horizon, (*a*) and the Hour Index will ſhew the Time of Day-break

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Part of it ſhall reſpect the South Part of the Heavens, the Situation of the Stars upon the Globe, with reſpect to the Points of the Compaſs, will alſo be ſimilar to that of the Stars in the Heavens at that Time; ſo that by comparing the Situation of the Stars upon the Globe, with thoſe in the Heavens, you may eaſily make yourſelf acquainted with all the Stars, that are viſible in this Part of the World.

If the Situation of the Moon or the other Planets is alſo to be repreſented, their Places in the Zodiac for that Time muſt be aſſigned them by Problem the laſt, their Longitude and Latitude being firſt found by an Ephemeris for that Purpoſe.

(*a*) Becauſe then the Place of the Sun will be ſo many Degrees below on the other Side, in which Situation of the Sun, we obſerved above, that the Twilight begins. The Quadrant of Altitude will ſhew when the Point oppoſite to that of the Sun is 18 Degrees above the Horizon. If this be tried for any Time while the Sun is paſſing from about the 5th Degree of *Gemini*, to the 20th of *Cancer*, that is, from about the 15th of *May*, to about the 7th of *July*, it will be found that there is no Beginning or End of Twilight, the Sun never being 18 Degrees below the Horizon (in the Latitude of *London*) during that Time.

OF

or the Beginning of Morning Twilight. And if the ſame Point of the Ecliptic be raiſed 18 Degrees above the Eaſt Side of the Horizon, the Index will point out the Time when the Evening Twilight ends.

## PROB. X. *To account for the Phænomena of the* Harveſt Moon.

About the autumnal Equinox, when the Moon is at or near the *Full*, it riſes almoſt at the ſame Hour for ſeveral Nights together; which Phænomenon is called the *Harveſt Moon*. (*a*) In order therefore to ſhew by the Globe how this comes about, let us ſuppoſe the Place of the Sun to be in the firſt Degree of *Libra*, as it always is at that Equinox, and that of the Moon to be in the Point oppoſite to the Sun, or the firſt Degree of *Aries*, becauſe the Moon muſt be conſidered as being at the *Full*. In this Caſe it is obvious that the Moon will riſe when the Sun ſets, which will be at Six of the Clock; becauſe both Luminaries are in the Equator and at oppoſite Points. Now ſince the Moon performs an entire Revolution in its Orbit, (which we will at preſent conſider as coincident with the Ecliptic) in the Space of one Month, it therefore advances forwards 12 Degrees each Day, and conſequently

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(*a*) Mr. *Johnſon* calls this Phænomena *Luna autumnalis*, and is the only Author that I know of that has accounted for it. Quæſtiones Philoſoph. Edit. 2da. Page 124.

the next Night it will be in the 12th Degree of *Aries*. Let us then enquire at what Time the Moon will riſe this Night. To this End, rectify the Globe to the Latitude of the Place, bring the Sun's Place, which is now the ſecond Degree of *Libra*, to the brazen Meridian, and put the Hour Index to 12; and if the Globe be turned till the Place of the Moon, *viz.* the 12th Degree of *Aries*, cuts the eaſtern Side of the Horizon, we ſhall find the Hour Index will point nearly to the Hour of Six, which, as obſerved above, was the time of the Moon's riſing the Evening before. And thus if we inveſtigate the Time of the Moon's riſing for ſeveral Nights together, both before and after the Full, it will be found much the ſame in all.

The Reaſon of this is, that about the Time of the full Moons, which happen at this Part of the Year, the Moon being in the aſcending Signs of the *Zodiac*, appears in its nocturnal Courſe to deſcribe a Parallel of the Equator one Night, much nearer the North Pole than another, and ſo riſes every Night more and more to the Northern Parts of the Horizon, and that conſiderably, as paſſing quite from the Southern to the Northern Tropic in a Fortnight's Time. Now it is eaſy to conceive that the nearer any of the Heavenly Bodies is to the North Pole, the ſooner that Body (*cæteris paribus*) aſcends the Horizon: Thus, if the Body be within a little of the *Circle of perpetual Apparition*, that Body is no ſooner ſet but it riſes again. In every Lunation therefore, while the Moon is paſſing from the Southern to the Northern Tropic, ſome Minutes are on that Account to be deducted from thoſe by which it would riſe later and later each Night, did the Plane of its Orbit coincide with that of the Equinoctial; and as much is to be added while ſhe paſſes from the Northern to the Southern Tropic. And for the like Reaſon in the former Caſe ſomething is to be added to the Time it would ſet at; and in the latter a like Quantity is to be deducted.

Now about the time of the autumnal Æquinox, the Sun being then in *Libra* or thereabouts, the Moon is in the midſt of the aſcending Signs, when at the *Full.* Hence it is that the *full* Moons at that Time of the Year, riſe ſo little later the one than the other.

In like manner, if any one thinks it worth while to obſerve it, he will find that the *new* Moons in the Spring, riſe nearly at the ſame Hour for ſeveral Nights ſucceſſively, while the full Moons ſhall riſe one later than another, by a greater Difference than at any other time of the Year. The Reaſon is, becauſe at the time of the new Moons, which happen at that Part of the Year, the Moon is in the aſcending Signs, but at the full, in the deſcending ones. (*a*)

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(*a*) It is here to be obſerved, that this Phænomenon is not the ſame as to Degree, in different Years; becauſe as the lunar Orbit is inclined to the Ecliptic 5 Degrees, and the Line of its Nodes is continually revolving round, the Inclination of that Orbit to the Equator will be ſometimes 10 Degrees greater than at others. So that the Moon does not haſten to the North, or deſcend to the South in each Revolution with equal Pace.

## Prob. XI. *To determine the Time of the Year in which a Star riſes or ſets* Coſmically, *or* Acronically.

Having rectified the Globe to the Latitude of the Place, bring the Star to the Horizon on the Eaſtern Side of the Globe, and obſerve what Degree of the Ecliptic riſes with it. Look for that Degree in the wooden Horizon, and right againſt it in the Kalendar, you will find the Month and Day when that Star riſes coſmically (*a*). And if you bring the Star to the Weſtern Side of the Horizon, the Degree of the Ecliptic which cuts the Eaſtern Side of the Horizon in that Situation of the Globe, will give the Day of the Month when the ſaid Star ſets coſmically (*b*). So likewiſe againſt the Degree which ſets with the Star, you will find the Day of the Month of its *Acronical* ſetting; and if you bring it to the Eaſtern Part of the Horizon, the Degree of the Ecliptic which then cuts the Weſtern Part of the Horizon will be the Sun's Place when the Star riſes *Acronically*.

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(*a*) For that will be the Day when the Sun and the Star riſe at the ſame time, which is its coſmical Riſing.

(*b*) The coſmical ſetting of a Star being its ſetting at Sun-riſe.

## PROB. XII. *To find the Time of the* Heliacal *riſing and ſetting of a* Star.

Having rectified the Globe to the Latitude of the Place, bring the Star to the Eaſtern Side of the Horizon, and apply the Quadrant of Altitude to the Weſtern Side in ſuch manner that its twelfth Degree may cut the Ecliptic, if the Star be of the firſt Magnitude (*a*). Then will the Point of the Ecliptic oppoſite to that which is cut by the Quadrant, be 12 Degrees below the Horizon. Look for this Degree on the wooden Horizon, and over-againſt it you will have the Day of the Year, when the Star riſes Heliacally. To find the Heliacal ſetting, bring the Star to the Weſtern Side of the Horizon, and turn the Quadrant of Altitude about to the Eaſtern Side, till the 12th Degree of it cuts the Ecliptic; then that Degree of the Ecliptic which is oppoſite to this Point, is the Sun's Place at that Time of the Year when the Star ſets Heliacally.

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(*a*) Becauſe a Star of the firſt Magnitude being above the Horizon, may be ſeen when the Sun is 12 Degrees below it. If the Star be of the 2d Magnitude, that Degree of the Ecliptic muſt be taken, which is 13 Degrees below the Horizon when the Star riſes, if of the 3d, then the 14th, *&c.*

## PROB. XIII. *The Day of the Month, the Time of the Day, and the Latitude of the Place being given, to find thereby the Hour of the Day, according to the* Babylonic, Italic, *and the* Judaical *Way of reckoning.*

1. To find the *Babylonic* Hour (which is reckoned from the Time of the Riſing of the Sun) having found the Time of Sun's riſing in the given Latitude, the Number of Hours between that and the Time given, is the Hour of the Day according to the *Babylonic* Way of Reckoning.

2. To find the *Italic* Hour (which is reckoned from Sun ſetting,) firſt find the Time at which the Sun ſets on the given Day. The Time elapſed from that to the Time given, will give the *Italic* Hour ſought.

3. To find the Hour according to the *Jewiſh* Way of Reckoning (one of which Hours is a twelfth Part of the Time that the Sun continues above the Horizon, and they are reckoned from Sun riſing,) firſt find the Hour the Sun ſets at, this being doubled will give the Number of Hours the Sun continues above the Horizon on the Day given. Then as that Number of Hours is to 12, ſo is the Number

of Hours ſince Sun riſing, to the *Judaical* Hour required.

Thus in the Latitude of *London*, when the Sun riſes at 4 of the Clock and ſets at 8; if the Hour given be 5 in the Evening, the *Babylonic* Hour will be the 13th, the *Italic* the 21ſt, and the *Jewiſh* Hour will be Nine and three Quarters.

## PROB. XIV. *To find the Place of any of the Heavenly Bodies upon the* Globe, *from their right Aſcenſion and Declination.*

Bring that Point of the Equator which expreſſes the right Aſcenſion given, to the brazen Meridian, and look upon the Meridian towards the North or South Pole, according as the Declination is North or South, for the Degree of Declination given, under which is the Place ſought (*a*).

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(*a*) For in this Caſe the Meridian is made to repreſent a Secondary of the Equator paſſing through the Place, an Arch of which lying between the Place and the Equator is the Declination, and the Diſtance between the Place of the Equator through which it paſſes, and the firſt Degree of *Aries*, is the right Aſcenſion.

# *Of the* Terreſtrial Globe.

**Prob. XV.** *From the Longitude and Latitude of a Place, upon the Surface of the Earth being given, to find the Place upon the* Globe.

This Problem is much the ſame with the laſt, only inſtead of the celeſtial, we uſe the terreſtrial Globe; and inſtead of reckoning the Longitude from the firſt Degree of *Aries*, the Point from which right Aſcenſion is reckoned, it muſt be counted from the firſt Meridian.

Bring the Degree of the Equator, which numbers that of the Longitude given, to the brazen Meridian, upon which count towards the North or South Pole, according as the Latitude is North or South, till you come to the Number of the Latitude, under which is the Place ſought. *(a)*

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*(a)* In ſome Globes, the Degrees of Longitude are numbered both Ways from the firſt Meridian, till they meet in the oppoſite Point, and the Longitude is diſtinguiſhed into Eaſt and Weſt as obſerved above: In ſuch Caſe it muſt be known which Kind the given Longitude is of.

## PROB. XVI. *To determine the Difference of Time in different Places.*

Find the Longitude of each Place, by the Converſe of the laſt Problem, and reduce the Difference into Time; which is done by allowing an Hour for every 15 Degrees, and ſo proportionably for leſſer Parts, the Number of Hours, *&c.* thus ſound, will be the Difference between the Time of the Day at one Place, and that at the other *(a)*.

## PROB. XVII. *To find the Diſtance between two given Places, and the Angle of Poſition, that is, the Bearings of them, or the Situation of one from the other, with reſpect to the Points of the Compaſs.*

Rectify the Globe to the Latitude of one of the Places, and bring that Place to the brazen Meridian; then fix the Quadrant of Altitude

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*(a)* The Reaſon of this is, becauſe as the Earth by turning round its Axis, brings all Places to the Sun once in 24 Hours; that Place which lies 15 Degrees to the Weſtward of another, comes to it an Hour later than the other; and ſo the Inhabitants of this Place are an Hour later in their reckoning of Time than at the other Place. And the contrary holds of ſuch as live towards the Eaſt.

to the uppermoſt Point of the Meridian, and putting the lower End of it between the Horizon and the Globe, ſlide it about till it paſſes through the other Place, and the Number of Degrees upon the Quadrant between Place and Place, turned into Miles by reckoning 60 to each Degree, will give the Diſtance between them; and the Number of Degrees upon the Horizon, between the Meridian and the Foot of the Quadrant, will give the Bearing of the Second, or its Situation from the firſt, with reſpect to the Points of the Compaſs.

## PROB. XVIII. *To find a Meridian Line, partly by the Help of the celeſtial Globe.*

Set the Hour Index to the Place of the Sun in the Ecliptic, and turn the Globe till the Pole Star, that is, the Star which is the neareſt the Axis of the Globe, comes under the Meridian, either above or below the Pole; above, if you find that the Index in that Situation of the Star, points to one of the Hours of the Night, otherwiſe below; and mark what Hour the Index points to. Then at that Hour of the Night hang up two Plummets, letting their Weights hang in Water to prevent their vibrating, in ſuch manner that the String of one of them may be directly between the Pole Star and the String of the other. Then will a Line drawn from one String to the the other, or the Shadow of the one when it falls upon the other, be a Meridian Line. And this I take to be a much more accurate, as well as an eaſier Method, than the common one of fixing up a Pin upon a Plane, drawing a Circle about the Foot of the Pin, and obſerving where the Extremity of the Shadow of that Pin enters the Circle in the Morning and goes out of it in the Afternoon, and drawing a Line from the Foot of the Pin through the middle Point between thoſe Places for a Meridian Line. For this latter Method requires more Care and Trouble to do it accurately, than they who have not tryed it, would imagine.

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## CHAP. XVI.

## *Of the* Equation *of* Time.

WHereas Time conſtantly paſſes on with an uniform and regular Flux, it is evident there is no poſſible Way accurately to meaſure and compare the ſeveral Intervals of it one with another, but by the Motion of ſome Body whoſe Progreſs is as uniform and regular as itſelf. To this End it was natural to pitch upon the Sun, whoſe Motion is obvious to all, and ſeemingly regular enough for that Purpoſe. But by the Circumſpection and accurate Obſervations of Aſtronomers, it is diſcovered not to be ſo; and that neither the Days, nor even the Hours themſelves, as meaſured by the Motion of the Sun, are of an equal Length. And this upon two Accounts, as will appear from the following Conſiderations.

I. A natural Day, or 24 Hours, is that interval of Time, in which the Sun ſeems to paſs from the Meridian of any Place to the ſame again: Had therefore the Earth no Motion but about its own Axis, and conſequently the Sun no apparent one along the Ecliptic, all the Days would be of an equal Length, as depending only on the Motion of the Earth about its Axis, which is uniform: But ſince the Earth is conſtantly moving in its Orbit, the ſame Way it turns about its Axis, the Sun is always in appearance advancing forwards. When the Earth therefore is turned round, and a Spectator upon its Surface brought again to the Place he was at the foregoing Day at Noon, it is not yet Noon with him, the Sun in appearance having advanced to the Eaſtward in the mean time; ſo that the Spectator muſt be carried farther ſtill, *viz.* ſo many Degrees as the Sun has advanced that Way, before he can be brought to the Meridian the Sun is in (*a*). Now by Reaſon of the unequable and

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(*a*) This is the Reaſon that the Length of the Day is not exactly equal to the Time, in which the Earth performs its Rotation about its Axis, which is, as was obſerved in Note (*f*), Chap. I. but 23 Hours 56 Minutes and 4 Seconds.

irregular Motion of the Earth in its Orbit, or, the irregular apparent Motion of the Sun along the Ecliptic depending thereon, the Number of Degrees the Sun advances Eaſtwards each Day, is ſometimes greater and ſometimes leſs; and conſequently, upon this Account, the Days muſt differ one from another in Length; and therefore the Hours alſo, ſince each Day is ſuppoſed to conſiſt of 24, be it long or ſhort.

II. Was the apparent Motion of the Sun through the Ecliptic uniform and regular, yet as the Ecliptic is oblique to the Equator, and therefore equal Portions of the one do not correſpond to equal Portions of the other; the Motion of the Sun, when referred to the Equator, would not be uniform; or, which is the ſame Thing, thoſe Points of the Equator which come to the Meridian with the Place of the Sun on different Days, would not be at equal Diſtances one from another; (*a*) which as to the Time of the Sun's appearing in the Meridian, or its ſhewing the Hour of the Day upon a Dial, is the ſame as if it had an irregular Motion along the Equator; and therefore muſt cauſe it to render the Days unequal among themſelves.

As theſe Cauſes are independent of each other, they ſometimes conſpire together; ſometimes they are contrary the one to the other.

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(*a*) This is eaſily tried upon a Globe, where if we bring every tenth Degree of the Ecliptic to the brazen Meridian, we ſhall find that each tenth Degree of the Equator, will not come thither with it.

The Time which an uniform Motion would mark out, is called *true Time*; and that which is indicated by the Sun, is *apparent Time*; and the Difference between theſe is termed *the Equation of Time.* When the abovementioned Cauſes have continued for ſome time, to render the apparent Motion of the Sun too quick or too ſlow, this Difference is conſiderable. About the 13th of *October*, it is the greateſt of all; at which Time the Sun is 16 Minutes and 11 Seconds too faſt.

## CHAP. XVII.

## *Of the* Diviſion *of* Time.

THE ſeveral Parts of Time are principally diſtinguiſhed by Days, Hours, Weeks, Months, Years, Cycles, Periods and Æras. Of which in their Order.

By a Day, according to the moſt natural and obvious Senſe of the Word, is meant that Space of Time during which it continues to be *Light*, and thus is oppoſed to that wherein it is *Dark*, which is called the Night. But this Space of Time being ſomewhat vague and indeterminate, the Time between the Sun's Riſing and Setting is uſually looked upon as the Day, and the Time which lapſes from its Setting to its Riſing again, the Night.

But the Word Day is frequently taken in a larger Senſe, ſo as to include alſo the Night, and to denote the Time of an whole apparent Revolution of the Sun round the Earth. In this Senſe it is called by ſome a natural Day, and by others an artificial one; but what ſome call the Artificial, others call the Natural. To avoid which Confuſion, it is uſual to call it in the former Senſe, ſimply the Day, and in the latter a *Nuchthemeron*; by which Term that Acceptation of it is aptly denoted, as being a Word that implies both Day and Night (*a*).

The *Nuchthemeron* is divided into 24 Parts, called Hours, which are of two Sorts, *Equal*, and *Unequal* or *Temporary*. The common Diviſion of the equal Hour is into Halves and Quarters: But Aſtronomers, and thoſe who are more accurate in their Account of Time, divide the Hour into ſixty Parts, called Minutes, and theſe again into ſixty Parts called Seconds, &c. And here we may obſerve, that the Word Minute is taken in a double Senſe, either to denote the ſixtieth Part of an Hour, and is therefore by Way of Diſtinction ſometimes called an *Horary* Minute; or elſe to ſignify the ſixtieth Part of a Degree, in which Caſe it may be called a *graduary* Minute, or Minute of a Degree.

Different Nations and People begin their Day at a different Hour. Thus the *Egyptians*

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(*a*) A Greek Compound of *νὺξ Night*, and *ἡμέρα Day*.

began their Day at Midnight; from whom *Hipparchus* introduced that Way of reckoning into Aſtronomy, and *Copernicus* and others have followed him. But the greateſt Part of Aſtronomers reckon the Day to begin at Noon, and ſo count 24 Hours till the Noon of the next Day, and not twice twelve, according to the vulgar Computation.

The Method alſo of beginning the Day at Midnight prevails in *Great-Britain*, *France*, *Spain*, and moſt Parts of *Europe*.

The *Babylonians* began their Day at Sun-riſing, reckoning the Hour immediately before its Riſing again, the 24th Hour of the Day. From whence the Hours reckoned in this Manner are called the *Babylonic*.

In ſeveral Parts of *Germany*, they begin their Day at Sun-ſetting, and reckon on till it ſets the next Day, calling that the 24th Hour. Theſe are uſually called the *Italian* Hours.

The *Jews* alſo began their *Nuchthemeron* at Sun-ſetting, but then they divide it into twice twelve Hours as we do, reckoning 12 for the Day, be it long or ſhort, and 12 for the Night (*a*). So that their Hours continually

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(*a*) According to this Way of reckoning, the Hours of the Day mentioned in Scripture are to be underſtood. The *Jews* alſo divided the Night into four Quarters, called *Watches*, each Watch containing three of their Night Hours. And diſtinguiſhed them ſometimes according to the Order of their Succeſſion. Thus we find in Scripture Mention of the *ſecond*, *third* and *fourth* Watch, ſometimes the firſt Watch was called the *beginning of the Watches*; the ſecond, the *middle Watch*, as not being ended till Midnight; and the fourth, the *Morning Watch*. The firſt alſo was ſometimes termed the *Evening*; the ſecond *Midnight*; the hird, the *Cock-crowing*; and the fourth, the *Dawning of the Day*.

varying with the Length of the Day and Night, the Hours of the Day were longer than thoſe of the Night one half of the Year, and the contrary the other. From whence their Hours are called *Temporary:* Theſe at the Time of the Equinoxes become equal, becauſe the Days and Nights are ſo then. The *Romans* alſo reckoned their Hours after this Manner, as do the *Turks* at this Day (*a*).

A Week conſiſts of the ſeven Days in moſt Countries, called after the Names of the Planets as above. But becauſe Eaſter Week

was

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(*a*) This kind of Hours are alſo called *Planetary*, becauſe the ſeven Planets were anciently looked upon as preſiding over the Affairs of the World, and to take it by turns each of theſe Hours, according to the following Order, *Saturn* firſt, then *Jupiter*, *Mars*, the *Sun*, *Venus*, *Mercury*, and laſt of all the *Moon*. Hence they denominated each Day of the Week from that Planet whoſe Turn it was to *reign* or preſide the firſt Hour of the *Nuchthemeron*. Thus aſſigning the firſt Hour of *Saturday* to *Saturn*, the ſecond will fall to *Jupiter*, the third to *Mars*, and ſo the twenty-ſecond of the ſame *Nuchthemeron* will fall to *Saturn* again, and therefore the twenty-third to *Jupiter*, and the laſt to *Mars*. So that on the firſt Hour of the next Day, it will fall to the Sun to preſide. And by the like Manner of reckoning the firſt Hour of the next will fall to the Moon, of the next to *Mars*, of the next to *Mercury*, of the next to *Jupiter*, of the next to *Venus*. Hence the Days of the Week came to be diſtinguiſhed by the Latin Names of *Dies Saturni*, *Solis*, *Lunæ*, *Martis*, *Mercurii*, *Jovis*, and *Veneris*; and among us by the Names of *Saturday*, *Sunday*, *Monday*, *Tueſday*, *Wedneſday*, *Thurſday* and *Friday*. For, as *Saturday*, *Sunday* and *Monday* plainly denote the Day of *Saturn*, the *Sun* and the *Moon*; ſo *Tueſday*, *Wedneſday*, *Thurſday* and *Friday*, denote the Day of *Tuiſco*, *Woden*, *Thor* and *Friga*, which are the *Saxon* Names reſpectively anſwering to *Mars*, *Mercury*, *Jupiter* and *Venus*. *Well*'s Chronol. Chap. II.

was formerly eſteemed the firſt or principal Week of the Year, and each Day thereof was a *Feria* or Holyday; hence the ſeveral Days were diſtinguiſhed among the primitive Chriſtians, by the Names of *Feria prima*, *ſecunda*, &c. that is, the firſt, ſecond Holyday, *&c.* But the Sunday, or *Feria prima*, was otherwiſe ſtiled by them the *Lord's Day*, as being the Day of our Lord's Reſurrection (*a*).

Months are diſtinguiſhed principally into two Sorts; *viz.* *Solar* and *Lunar*. The Solar Month is either *Aſtronomical* or *Civil* (*b*). The Lunar Month is divided into *Periodical*, *Synodical* and *Civil*. The *Aſtronomical* Solar Month is the Time the Sun takes up in paſſing through one of the Signs of the Ecliptic.

The *Periodical* and *Synodical* lunar Months have already been obſerved; the former to be equal to the Revolution of the Moon about the Earth, *viz.* 27 Days 7 Hours and 43 Minutes; the latter, the Interval of Time that lapſes between two ſucceeding Conjunctions of the Moon with the Sun, *viz.* 29 Days, 12 Hours, 44 Minutes and 3 Seconds.

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(*a*) The Hebrew Word which in the old Teſtament is rendered *Week*, ſignifying a Collection of ſeven, is there ſometimes uſed for ſeven Years; and in Conformity thereto, our *Engliſh* Word *Week* in thoſe Places muſt be underſtood to imply not a *Week of Days*, but a *Week*, or *Syſtem of ſeven* Years.

(*b*) A Month or Year is denominated *Civil*, from its being of common Uſe in any Republic or Society of People.

Now becauſe in common Uſe it would be inconvenient to have the Months to conſiſt of odd Pieces of Days; it was uſual among the *Jews*, *Greeks* and *Romans*, (till *Julius Cæſar* made an Alteration) in order to adapt their Months to the apparent Motion of the Sun, to have their Months conſiſt of 29 and 30 Days alternately, one of theſe Numbers being about half a Day above, and the other about half a Day leſs than the Synodical Month; ſo that by this Means the ſame Appearances of the Moon would happen nearly on the ſame Day of the Month for a long Time together. This Month thus adapted to the laſt mentioned lunar Month, is called the *Civil* lunar Month; and is ſtill made uſe of by the *Turks*.

The *Egyptian* Months conſiſted of 30 Days each, 12 of theſe make a Year wanting 5 Days, which they added to the End of the Year, and from thence called them Epagomenæ, or Supplimental.

Now becauſe 12 civil lunar Months want 11 Days of a Year, it is evident that the ſeveral Seaſons of the Year muſt in Time fall upon different Months; to prevent which, *Julius Cæſar* ordained, that the Month ſhould not be taken from the Motion of the Moon, but from that of the Sun; that they ſhould conſiſt alternately of 30 and 31 Days, and that *February* ſhould contain 29, and every fourth Year 30. But it happened afterwards that the Name of the ſixth Month, which was formerly

*Sextilis*, was in Honour of the Emperor *Auguſtus* changed for that of his, as that of the foregoing Month *Quintilis* had before been, in Honour of his Father *Julius*; and therefore ſince this Month conſiſted but of thirty Days according to the Inſtitution of *Julius Cæſar*, one more was added to it; to make it equal to the foregoing one, that the Honour paid to the Son might not ſeem to fall ſhort of that which was paid to the Father. Upon this, the next Month, *viz. September*, was reckoned but 30, the next 31, and ſo on; that the alternate Order abovementioned might not be deſtroyed, till *January*, which, according to that Order, ought to have had but 30. But this Month being ſo named in Honour of *Janus*, the God of Time, it was though improper to rob the God of a Day, wherefore *February* was reduced to 28, and every fourth Year to 29. And ſo the Months came to ſtand as they are in our Calendar, from whence they are called *Calendar* Months. Theſe are properly *civil Solar Months*.

The principal Diviſion of the Year is alſo into two kinds, *viz. Solar* and *Lunar*; each of which are again divided into *Aſtronomical* and *Civil*.

The Aſtronomical Solar Year is of two kinds, *viz. Tropical* and *Sydereal*. The former is the Space of Time which lapſes while the Sun appears to move from either of the Soltitial or

Equinoctial Points to the ſame again. Now by Reaſon of the continual Preceſſion of the ſaid Points, whereby they do as it were meet the Sun, this kind of Year is ſomething leſs than the *Sydereal* one; which is the Time in which the Earth performs an entire Revolution in its Orbit, or in which the Sun ſeems to paſs from any fixed Star to the ſame again. The former conſiſts of 365 Days, 5 Hours, 48 Minutes, and 57 Seconds; the latter of 365 Days, 6 Hours, 9 Minutes, and 14 Seconds, and is alſo called *Periodical*, or *Anomaliſtical*. The Aſtronomical Lunar Year conſiſts of 12 Synodical Months; and is therefore 354 Days, 8 Hours, 48 Minutes, and 36 Seconds.

The *Civil* Lunar Year is alſo of two kinds, *viz. moveable* and *fixed*; the moveable civil Lunar Year conſiſts of 12 civil Lunar Months, which being but 354 Days, falls ſhort of a Year by 11 Days, ſo that the beginning of this Year varies with reſpect to the Seaſons, till it runs through them all, which it does in about 32 Years Time; and from hence it is denominated *Moveable*, or the *wandring Lunar Year*; As the *Turks* now make Uſe of the civil Lunar Month, ſo they alſo do of this civil Lunar Year.

In order to render this kind of Year *fixed*, that is, to prevent the Seaſons from ſhifting from one Part of it to another, the *Greeks*, and the *Romans*, till *Julius Cæſar*'s Time, after every three Years added a Month of thirty-three Days, which they called *Embolimæus*, or the *Intercalary* Month.

But becauſe this Method is not ſufficient, (for the civil Lunar Years fall ſhort of the Aſtronomical Solar Years above 11 Days,) others adapt ſuch Intercalatians to them as do in a manner render the beginning of them fixed to the ſame Part of the Solar Year. Hence theſe Years properly intercalated are called *fixed* Lunar Years, as alſo from their Dependency on the Motion both of the Sun and Moon, *Luni-Solar* Years. Years of this kind are uſed by the *Jews*, and the Clergy of the Church of *Rome* in their Eccleſiaſtical Affairs.

The civil Aſtronomical Year is alſo either *moveable* or *fixed*. The moveable one was in Uſe among the *Egyptians*, and is from thence called, the *Egyptian* Year. It conſiſted of 365 Days, and therefore fell ſhort of the tropical Year, with which all the Seaſons return, near ſix Hours; ſo that four ſuch Years are leſs than four Aſtronomical Solar ones, by almoſt a whole Day, and therefore in 1460 Years, the beginning of the Year would *paſs* through all the Seaſons.

Although the *Romans* did not uſe this kind of Year, yet theirs, taking one Year with another, was of the ſame Length (*a*); *Julius Cæ-*

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(*a*) For having as obſerved above, ſix Months of 29 Days each, and ſix of 30, and an *Intercalary* Month of 33 Days, at the End of every third Year; their Years conſiſted one with another of 365 Days.

*ſar* therefore to *fix* this Year, when he rectified the Calendar, and threw out the civil Lunar Months, to make up the ſix Hours which this Year wants of the true Aſtronomical Solar Year, and with which only the Seaſons keep pace, ordered that every fourth Year ſhould have an intercalary Day, *i. e.* that a Day ſhould be added to the Calendar, over and above thoſe it contain'd the other three, or that every fourth Year ſhould contain 366 Days. Hence it was that as we obſerved above, *February* came to have 29 Days every fourth Year. But he did not add it to the End of the Month, but ordered that the ſixth of the Calends of *March*, which according to our way of Reckoning is the 24th of *February*, ſhould be reckoned twice. Upon which Account that Year was wont to be called *Biſſextilis*, and is ſtill retain'd by us under the Name of *Leap* Year. From hence, Years reckoned after this Manner are called *Julian* Years.

But however the Year was not rendered *fixed* by this means, for it was now too long by about eleven Minutes, and therefore varied from the Sun about a Day in 131 Years. So that from the Time of the *Nicene* Council, (in which the Rule was eſtabliſhed for the Time of celebrating *Eaſter*) to the Year 1582, in which Pope *Gregory* the XIIIth undertook a farther Reformation of the Calendar, there was found to have roſe a Variation of 10 Days, the vernal Equinox being obſerved to fall on the 11th of *March*, which at the Time of the *Nicene* Council, fell on the 21ſt. The Pope therefore in order to bring back the Equinoxes to the Time of the Year they fell upon at the Time of that Council, took thoſe ten Days out of the Kalendar by ordering the 5th of *October* 1582, to be reckoned the 15th, which neceſſarily removed the following Equinox from the 11th to the 21ſt of *March*. And to prevent the Seaſons of the Year from going backwards for the future, he ordered that every hundredth Year, which according to the Form inſtituted by *Julius Cæſar*, was to be a *Biſſextile* or *Leap* Year, ſhould be a common one, and conſiſt only of 365 Days, but that ſince too much would be taken away by this Means, every fourth hundred Year ſhould remain *Biſſextile*.

This Method being eſtabliſhed by Pope *Gregory*, is called the *Gregorian* Account, or the *New Stile*, as being new in compariſon of that eſtabliſhed before by *Julius Cæſar*, which is now called the *Old*; and is obſerved in all Places where the Papal Authority is acknowledged; as alſo, towards the End of the laſt Century was received by many of the reformed People in *Germany*. In *Great-Britain* and *Ireland*, and in ſome reformed Nations of the Northern Parts of *Europe*, the old *Julian* Form is ſtill retained. But above an hundred Years being now lapſed ſince the abovementioned Reformation of the Calendar, and the hundredth Year not being made a common one, as in the *Gregorian* Method; the *Gregorian* Account differs now from the *Julian* one Day more than it did at its firſt Inſtitution; ſo that that Day which according to the *Old Stile* is the firſt of the Month, is the 12th of the ſame according to the *New*.

As the Form of the Year in different Nations is various, ſo alſo is the Beginning of it. Thus the *Jews* begin their Eccleſiaſtical Year with the *new* Moon of that Month whoſe *full* Moon happens next after the vernal Equinox. The Church of *Rome*, with the *Sunday* that falls upon the ſaid *full* Moon, or that happens next after it; that is, with the Feaſt of the Reſurrection of our Lord. The *Grecians* began theirs with the new Moon that happened next after the Summer Solſtice. And the *Romans* anciently began theirs with the new Moon next after the Winter Solſtice. The *Venetians*, *Florentines*, *Piſans* in *Italy*, and the Inhabitants of *Triers* or *Treves* in *Germany*, make the *vernal* Equinox the Beginning of their Year. The ancient Clergy made the 25th of *March*, or the Feaſt of the Bleſſed Virgin, the Beginning of the Year; which Method the Church of *England* ſtill retains; and according to our Law the Year begins here too, though in Things that do not require a *legal Date*, we, with the neighbouring Nations, look upon the firſt Day of *January* to be the Beginning of the Year.

The *Perſians* ſtill retain the *Egyptian* Year, and the Period in which the firſt Day of it paſſes through all the Seaſons of the Aſtronomical Solar Year, *viz.* 1460 Years, they call the *Sothiacal Period*, or *great Dog-Star Year*; as being reckoned to begin in that Year in which the *Dog-Star* (or *Sothis*) riſes Heliacally on the firſt Day of the Month *Thoth*, which is the firſt Day of their Year.

A *Cycle* is a continual and ſucceſſive Revolution of a certain Number of Years. The Cycles are four, *viz.* the Cycle of the Sun, the Cycle of the Moon, the Cycle of *Dionyſius*, the Cycle of the Indiction.

In order to apprehend the Foundation and Nature of the Cycle of the Sun, it muſt be obſerved that to each Day of the Year is aſſigned in the Calendar, ſome one of the firſt ſeven Letters of the Alphabet A, B, C, D, E, F, G, the firſt Letter A being affixed to the firſt Day of *January*, to the ſecond B, and ſo to G. After which the Letters are repeated again in the ſame Order, A falling to the 8th, B to the 9th, *&c.* By which means, becauſe the common Year contains 52 Weeks and one Day over, the laſt Day of *December* has alſo the Letter A affixed to it. The Letters being appropriated in this manner to each Day of the Year, it follows, that which ever of the Letters belongs to any particular Day in the firſt Week of *January*, that Letter will belong to the ſame all the Year long. But ſince the ſame Letter, *viz.* A, is affixed to the laſt, which is affixed to the firſt Day of the Year, it is obvious, that at the End of one Year and beginning of the next, two ſucceeding Days will have the ſame Letter A; ſo that if *Monday* had an A the laſt Year, *Tueſday* has it this, and *Monday* the G: And therefore the Letters will move backwards with reſpect to the Days of the Week; for Inſtance, that Day which has G one Year, will have F the next, after that E, then D, and ſo on to A; and conſequently the ſame Day will run through the ſeven Letters in ſeven Years Time. But before the ſeven Years are expired, a Leap Year will intervene, having one Day more than common, which Day is added at the 24th of *February* (*a*), and has the ſame Letter that the 24th has, ſo that in this Caſe, as there was a Change or Regreſſion of the Letters, with reſpect to the Days of the Week, upon the Account of two Letters coming together at the End of one Year, and the beginning of the next, there will be another like Change in *February* alſo; and therefore, as in the former Caſe, the ſame Letter would have run through all the Days of the Week in ſeven Years; this ſecond Change happening once in four

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(*a*) Some ſay it is added after the 28th, and that it has the ſame Letter with that. From this Uncertainty there ariſes a Difficulty in fixing the Day of the Celebration of the Feſtival of St. *Matthias*, which has occaſioned a great Diſpute among the Learned.

Years, the Courſe of the Letters is ſo oft interrupted, that it does not become the ſame again, till after four times 7, or 28 Years.

Thus, ſuppoſe a Leap Year begins with a *Sunday*, it will end with a *Monday*; in which Caſe, A and G will be the *Dominical* or *Sunday* Letters, becauſe A is appropriated to the firſt of *January*, and G to the 30th of *December*: But ſuch another Year cannot return till after 28 Years, as any one that tries will readily find; the Conſequence of which is, that thoſe two Letters cannot be *Sunday* Letters again in one Year, till after that ſpace of Time. And becauſe the ſame *Sunday* Letters do return after every 28 Years, that Term is called the *Cycle of the Sunday Letter*, or otherwiſe, though not ſo properly, *the Cycle of th Sun* (*a*).

The Cycle of the Moon is a Revolution o 19 Years; after which Time, as was anciently ſuppoſed, the new and full Moons would fall upon the ſame Days of the Year again (*b*).

This Cycle is therefore of great Note in the

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(*a*) The ninth Year before our Saviour's Birth, was the firſt of this Cycle; from whence may eaſily be known what Year of it the preſent is. The firſt of this Cycle is always Leap Year, and has for its Dominical Letters G and F. And the Letter A is as already obſerved, always affixed to the firſt Day of *January*. From which *Data* may at any Time be computed what is the *Sunday* Letter for the preſent Year, and from thence what Day of the Week each Day of the Year falls on.

(*b*) This is alſo called the *Metonic* Cycle from *Meton*, the Author of it.

Chriſtian Church, a Method founded upon this Suppoſition, being eſtabliſhed by the Fathers of the *Nicene* Council, for finding the Time of celebrating *Eaſter*, and the other moveable Feaſts of the Church for any Time to come.

In order to underſtand which, we muſt obſerve, that as the Chriſtian Paſſover, or Feſtival of *Eaſter*, ſucceeded the *Jewiſh* one, the Time of its Celebration was regulated by that upon which the *Jewiſh* Paſſover was at its Inſtitution, by God himſelf appointed to be kept, which was on the fourteenth Day of the firſt Month, according to the manner of reckoning among the *Jews*, *Exod.* 12. Now the *Jewiſh* Months being Lunar ones, each Month began upon the Day of the new Moon, or however at the Time of its *Heliacal* riſing, and ſo the full Moon fell upon the fourteenth of each Month. And further, that Month was called the firſt of the Year, whoſe full Moon or fourteenth Day, either fell upon the vernal Equinox, or was the firſt that ſucceeded it. And at the Time of the *Nicene* Council, when the Rule for finding *Eaſter* was drawn up, the Equinox fell, or was thought to fall, on the 21ſt of *March*. But becauſe our Saviour's Reſurrection, which the Feſtival of *Eaſter* is deſign'd to commemorate, happened on a *Sunday*; it was ordered that it ſhould not be kept on the full Moon, but the *Sunday* following. Accordingly the Rule then eſtabliſhed for finding *Eaſter*, and which is ſtill made uſe of by the Church, runs thus; *Eaſter-Day is always the firſt* Sunday *after the firſt full Moon, which happens next after the One and twentieth Day of* March. *And if the full Moon happens upon a* Sunday, Eaſter-Day *is the* Sunday *after.*

The Foundation of the Rule being thus explained, it remains now to ſhew the Meaning of it, which is not ſo obvious as at firſt Sight it appears to be.

It is to be obſerved then, that in order to find on what Days of the Year the new and full Moons would happen for the Time to come, the ancient Method was to obſerve on what Day of each Month, the new Moons fell in each Year of the Moon's Cycle, and to the ſaid Days reſpectively to ſet the Number of the Year in their Calendar. Thus, obſerving that the new Moons in the firſt Year of the Cycle fall on *January* the 23d, *February* 21, *March* 23, *&c.* to thoſe Days they affixed the Number 1. And in like manner obſerving, that, in the ſecond Year, they fell on *January* 12, *February* 10, *March* 12, *&c.* to each of them they put the Number 2; and ſo for each Year of the Cycle. And on Account of their great Uſefulneſs, or becauſe they were wrote in Letters of Gold, the Number thus ſet to the Days on which the new Moons fell, were called the *Golden Numbers* (*a*).

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(*a*) In the Table for finding *Eaſter* for ever in ſome Common Prayer-Books, they are ſtiled Prime Numbers, probably as being placed in the Prime or firſt Column of the Calendar.

The Numbers being thus affixed in the Calendar, it is only looking for the Golden Number of any Year; and over-againſt it you have the Day in each Month of that Year, in which the new Moon happens.

Thus, at the Time of the *Nicene* Council the Time of the new Moon's might be accurately enough found, but not at this Time, becauſe the new Moons do not happen at the ſame Time of the Year every 19 Years, but fall ſhort an Hour and half, which in 304 Years comes to a Day, ſo that now the new and full Moons fall almoſt 5 Days ſooner than they are ſhewn to do by the Golden Numbers, affixed to the Calendar.

However, as no proper Authority has interven'd to alter the Method of finding the *Eaſter* full Moon by the Golden Numbers, eſtabliſh'd at the *Nicene* Council, that Method is ſtill retained in the Church. And therefore to underſtand the abovementioned Rule aright, it muſt be ſuppoſed, that by the full Moon is meant the Time of the full Moon as found by the Golden Numbers affixed to the Calendar in the Common Prayer Book, and not by an Almanac, or by Aſtronomical Obſervation (*a*).

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(*a*) The firſt Year of Chriſt was in the 2d of this Cycle. By the abovementioned Method the true new Moon may at any Time be found, if we deduct five Days in Conſideration of the abovementioned Deficiency in the Moon's Cycle, and ſuppoſe the new Moon to fall five Days ſooner than that Method ſhews it to do.

By multiplying the Cycle of the Sun and Moon together, we have a third Period, containing the Properties of both the other two, ſo that at the End of this Cycle not only the Days of the Week have the ſame Letters belonging to them, and ſo fall on the ſame Days of the Month, but the new and full Moons alſo fall on the ſame Days of the Year, ſo that the Time of celebrating *Eaſter* and the other Feſtivals of the Church, return to the very ſame Days of the Year. From whence this is called *the great Paſchal Cycle*, and ſometimes from its Author, the *Dionyſian Cycle* (*a*). It conſiſts of 532 Years, and the firſt Year of it was 457 Years before Chriſt.

The *Cycle of the Indiction* is a Revolution of 15 Years made uſe of by both the *Greeks* and *Romans*, probably upon ſome political Account. It has no Relation to the Motions of the Heavenly Bodies. The Nativity of our Saviour fell upon the 4th of this Cycle.

By multiplying the three Cycles of the Sun, Moon, and Indiction together, ariſes a Term of 7980 Years, called the *Julian Period*; which as it began ſeveral hundreds of Years before the Creation, and is not now ended, and in all probability will not, ſo long as the World ſtands, is of ſingular Uſe in Chronology, as

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(*a*) This is alſo by ſome called the *Victorian* Period.

taking in all Events, both paſt, preſent and to come.

This Period was invented by *Joſeph Scaliger*, and began 4713 Years before Chriſt.

An *Æra*, otherwiſe called an *Epoch*, is a Continuation of Time beginning from ſome certain Point, as from a Root, and continually proceeding forwards without beginning again.

That which is of principal Note among Chriſtians, is the Æra of Chriſt.

The Author of this Æra was *Dionyſius Exiguus*, who flouriſhed about 500 Years after Chriſt. He began it on the Annunciation of the Virgin *Mary*, or the 25th of *March*: the 25th of *March*, as he ſuppoſed, immediately preceeding the Nativity of our Lord: but it is now generally thought that our Saviour was born the *December* before that, wherefore others, beginning the Æra from the Nativity it ſelf, or the 1ſt of *January* following it, reckon almoſt a quarter of a Year before thoſe who adhere to the *Dionyſian* Account.

The *Engliſh* and *Iriſh* ſtill adhere to the *Dionyſian* Account in their eccleſiaſtical and civil Affairs, which all the reſt of the World, and even they themſelves in common Account, have laid aſide for that which begins at *January* the firſt. This Æra is frequently reckoned backwards, as well as forwards: Thus, it is uſual to ſay an Event happened ſo many Years before Chriſt.

The firſt Year of this Æra, agreeably to what was obſerved above, anſwers to the 4714th of the *Julian* Period.

There are ſeveral *Epochs* or Æra's of leſs Note, which I ſhall but juſt mention; this Chapter having already run out to a much greater Length, than I at firſt intended.

The Æra of the *Creation* according to the *Greek* Church, and which is now of Uſe in ſome Eaſtern Nations, begins before the *Julian* Period, *viz.* 5508 Years before Chriſt. Tho' others, among whom are the greateſt Aſtronomers, begin it ſomething leſs than 4000 Years before our Saviour's Birth.

The Æra of the *Deſtruction of Troy*, according to *Dionyſius Halicarnaſſeus* and *Diodorus Siculus*, begins 1181 Years before Chriſt.

The Æra of the *Olympiads*, or *Olympic Games* of Uſe among the *Greeks*, 776 before Chriſt.

The Æra *Urbis conditæ*, or *of the Building of* Rome, made Uſe of by the *Romans*, according to *Varro*, began 753 Years before Chriſt; according to the *Faſti Capitolini*, 752.

The Æra of *Nabonaſſar* made Uſe of by the *Chaldeans* and *Egyptians*, famous among Aſtronomers, as conſiſting of *Egyptian* Years, which are diſturbed by no Intercalation. It begins 747 Years before Chriſt.

The Æra of the *Death of Alexander the Great*, 324.

The Æra of the *City of Antioch*, 49.

The Æra of the *Julian Reformation of the Calendar*, 45.

The Æra *Actiaca*, 30.

The *Diocleſian Æra*, 284 Years after Chriſt.

The Æra of the *Hegira*, or *Flight of Mahomet*, uſed by the *Turks* and *Arabs*, 622 after Chriſt.

The Æra of *Yeſdegird* or *Perſian Æra*, 632 after Chriſt.

*N. B.* A Year in the Æra's of the Death of *Alexander*, and of *Nabonaſſar*, conſiſts but of 365 Days; and a Year of that of the *Hegira* is only 354.

As the Subject of this Chapter, though founded in *Aſtronomy*, relates principally to the Science of *Chronology*, of which this *Compendium* is not deſigned to treat; I have little more than thrown a few Definitions together, leaving it to thoſe who would ſee more on the Subject, to conſult ſuch Authors as have wrote more largely thereon; as *Gregory*, *Keil*, *Wells*, *Whitby* on the Common-Prayer, and many others.

# CHAP. XVIII.

## *Of the* Forces *neceſſary to retain* Bodies *revolving in* circular *and* other Orbits.

### *And firſt of* circular Orbits.

IN Order that a Body may move round in the Circumference of a Circle, it is requiſite, that as ſoon as the Body begins to move, ſome Power or Force continually act upon it, in ſuch manner as to make it bend its Courſe every Moment towards the Center of the Circle; becauſe, as is obvious to conceive, it would otherwiſe only move right forwards (*a*); for in Conſiderations of this kind, we do not ſuppoſe the revolving Body to be affected even by its own *Weight*, or any other moving Cauſe whatever, except that whereby its Motion is made circular: And becauſe the Circumference of a Circle is every where equally diſtant from the Center thereof, it is obvious there muſt be the moſt exact *Adjuſtment* imaginable between the Power that retains the Body in the Circle, whatever that Power be, and the Endeavour the Body has to move right on in a ſtraight Line: For in Caſe the former be an *over-balance* for the latter, it will bring the Body *nearer*

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(*a*) See Part I. Chap. IV.

and *nearer* to the Center of the Circle it ſhould deſcribe; and on the other hand, if it be too *weak*, it permits it to move *farther* and *farther* off. So that the Body, inſtead of a Circle, will deſcribe ſome other Figure, which will be of this or that Form, according as the Force the Body moves with, compared with the Power which bends its Courſe, is greater or leſs; and not only ſo, but as this latter Force (which is ſometimes the Caſe) acts more or leſs powerfully, as the revolving Body comes nearer or goes farther from the Point it revolves about. And here we may have recourſe to a Fact obvious to every one, which will in ſome Meaſure illuſtrate what we have been ſpeaking of. Let a Ball be hung up by a String: This Ball, when drawn a little to one Side, will endeavour to return back to the loweſt Point, as if there were ſome Power ſeated in that Point to draw it thither from all Sides. But the Body may have a Caſt given it ſideways in ſuch Manner, as that by Virtue of the Caſt, and by Virtue of its Tendency to the loweſt Point together, it ſhall deſcribe a Circle parallel to the Horizon; or inſtead of that it may be made to deſcribe other Figures, as Ellipſes ſuppoſe, which will be of various Forms according to the Force and Direction of the Caſt given to the Ball. And if we ſuppoſe ſeveral Balls hung up, which when removed to *equal* Diſtances from their loweſt Points, ſhould tend thither again with *unequal* Forces, and theſe

Forces urging them by different Laws and in different Manners, the Figures deſcribed by them, when thrown Sideways with this or that Force and Direction, will be proportionably different one from another. Now in this Suppoſition, the Caſt which is given to the Ball Sideways, correſponds with the Endeavour a Body revolving in the Circumference of a Circle or other Figure, has to move right on continually, and ſo to leave the Center it ought to revolve about; and the Endeavour, Struggle or Tendency the Ball when drawn to one Side, has to move towards the loweſt Point, or rather the Power which is, as it were, ſeated in that loweſt Point, and occaſions the abovementioned Endeavour or Tendency in the Ball to move thither, correſponds with the Power we ſuppoſed above neceſſary to retain a Body in the Circumference of the Circle or other Figure it deſcribes, and which by bending the Courſe of the Body inwards, that is, towards the Center of the Figure, prevents it from going off in a ſtraight Line.

We are therefore in the next Place, to enquire what Forces or retaining Powers are neceſſary to bend the Courſe of Bodies in ſuch Manner that they ſhall deſcribe this or that Orbit (*a*); and ſince the Planets both primary

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(*a*) For unleſs we can *meaſure* the Forces we aſſign, and ſay they are exactly equal to the Effects we ſuppoſe them to produce, it would be in vain to aſſign them at all. And this is what makes the Difference between true and falſe Philoſophy.

and ſecondary, as alſo the Comets, deſcribe Orbits either circular or elliptical, as obſerved above, we will confine ourſelves chiefly to the Conſideration of the Forces neceſſary to retain revolving Bodies in ſuch Orbits. This Subject has been already touched upon in the firſt Part of this *Compendium* (*a*), under the Title of *Centripetal and Centrifugal Forces*; but little more is taken Notice of there, than what was requiſite to ſhew the Poſſibility of ſuch a Motion as we are ſpeaking of, and to explain the Terms; which are theſe that follow, *viz. Centripetal* Force, or that whereby the revolving Body is prevented from going on in a right Line: The *Centrifugal* Force, or that by which it endeavours to go on in ſuch a Line. Theſe, like Action and Reaction, are equal in all Caſes, and therefore called by one common Name, *Central* Forces: That whereby the Body continues its Motion, is the *Projectile* Force; and the Time in which it revolves once

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The former by a mathematical Scrutiny, (as it were by a kind of Menſuration) finds that the Cauſes aſſigned are juſtly adapted to the Facts accounted for; while the other, by a conjectural kind of Method, aſſigns Cauſes which perhaps are plauſible at firſt Sight, but when meaſured by the Rules of Geometry, are found either too ſmall or too great for the Effects they are ſuppoſed to produce.

This was the Caſe of *Des Cartes*'s Vortices, the Doctrine of which captivated the whole Philoſophical World, and continued to do ſo, till they were weighed in that unerring Balance, and thereby diſcovered to be inadequate to the Purpoſes for which they were contrived.

(*a*) Chap. VIII.

round, is called the *Periodical Time.* The ſeveral Caſes proper to be conſidered, ſhall be compriſed in the following Propoſitions.

## PROPOSITION I.

When two or more Bodies revolve at *equal* Diſtances from the Center of the Circle they deſcribe, but with *unequal* Velocities, the central Forces neceſſary to retain them, will be to each other, as the Squares of their Velocities. That is, if one revolves twice as faſt as the other, it will require four times the retaining Force the other does; if with three times the Velocity, it will then require nine times the Force to retain it in its Orb. *&c.* (*a*)

*N. B.* We do not here conſider the Magnitude of the revolving Body, becauſe we ſuppoſe the Power at the Center, to act upon every Part of it alike, ſo that the larger it is, the more forcibly in Proportion it is acted upon; and therefore it is the ſame thing, whether it be large or ſmall.

*(a)* Let BDF Fig. 25. repreſent a Circle whoſe Center is S, draw ABC a Tangent to the ſame at the Point B, and let there be a Body moving from A towards C, and let it be ſuppoſed to be deſtitute of Weight, or whatever elſe might alter its Courſe. In this Caſe it would deſcribe the right Line ABC; but when it comes at B, let it begin to be acted upon by ſome central Force, or retaining Power ſeated in S, ſuch as ſhall *bend* its Courſe at B from a rectilineal to a circular one, and acting upon it afterwards in every Inſtant of Time in like manner, ſhall cauſe it to deſcribe the Circle BDFB. Let us then proceed to eſtimate the Quantity of Force exerted in producing one of theſe *Bendings*; in order to which let us conceive a Point of the Circle as D to

be the very next to the Point B, though taken at ſome Diſtance in the Figure to avoid Confuſion of Lines. Parallel to the Diameter BF draw the Line CD, and Parallel to the Tangent BC the Line DG, draw alſo the Line BD, which will be the Diagonal of the Parallelogram CG. Now it is well known that ſuppoſing a Body would move from B to C, and by Virtue of ſome other Force acting upon it at the ſame Time, would move from B to G, it will neither move to C nor G, but to D; that is, its ſtraight Courſe will be *bent* into a circular one at B, for that is what is meant by its moving from B to D when B and D are contiguous, as is here ſuppoſed. BG therefore is the Space over which the revolving Body would move by that Action of the central Force which cauſes the Bend at B, or any other Point of the Circle. And as BG is equal to the Space over which the revolving Body would move by Virtue of the central Force, it will always be proportional to the Force it ſelf. The Value of that Line is therefore now to be determined. Join the Points D and F, then (by 31. El. 3.) the Triangle BDF will be rightangled at D, and conſequently (by 8 El. 6.) BG : BD : : BD : BF, from whence $BG = \frac{BDq}{BF}$; but when B and D are contiguous (as is here ſuppoſed) the Difference between the Chord BD and the Arch BD vaniſhes, ſo that $BG = \frac{\text{the Arch } BDq}{BF}$ Having thus found a Quantity proportionable to the Act of the central Force requiſite to retain a revolving Body in a Circle, we may proceed to eſtabliſh the following fundamental *Lemma*.

## LEMMA I.

The Space that a Body retained in a Circle, by Virtue of any central Force, would move over in a given Time, by Virtue of that Force only, is equal to the Square of the Arch deſcribed in that Time, divided by the Diameter of the Circle.

*Dem.* The Action of a central Force upon a revolving Body is analogous to that of Gravity upon falling Bodies, and therefore the Space a Body would move over by Virtue of that Force only, is as the Square of the Time that Action continues: as then the Square of the Time the Arch BD is deſcribed in (the Points B and D being contiguous as before) is to $\frac{BDq}{BF}$ which by what was ſhewn above, is a Quantity equal to the Space the Body would move over towards the Center in that Time, ſo is the Square of the Time given to the Space the Body would move over by Virtue of that Force in that Time. If we then put *t* to expreſs the Time the Arch BD would be deſcribed in, T for the Time given, and S the Space the Body would move over towards the Center in that Time, we ſhall have this Proportion.

| | | |
|---|---|---|
| *Viz.* | 1 | $tq : \frac{BDq}{BF} :: Tq : S$ |
| Let the Arch the Body would deſcribe in the Time T, be the Arch BN, then becauſe the Motion of the Body in the Circumference of the Circle is uniform, we have | 2 | $t : T :: BD : BN$ |
| Squaring the laſt Proportion, we have | 3 | $tq : Tq :: BDq : BNq$ |
| Altering the Poſition of the Means in the firſt Proportion, we have | 4 | $tq : Tq :: \frac{BDq}{BF} : S$ |
| Comparing the third and fourth Step together. | 5 | $BDq : BNq :: \frac{BDq}{BF} : S$ |
| Turning the 5th Step into an Equation. | 6 | $S \times BDq = \frac{BNq \times BDq}{BF}$ |
| Dividing the laſt by BDq | 7 | $S = \frac{BNq}{BF}$ Q. E. D. |

*Coroll.* From this *Lemma* it follows, that the central Force requiſite to retain a revolving Body in the Circumference of a Circle, muſt be ſuch as ſuppoſing the projectile Motion ſtopp'd, would cauſe the Body in the Time it would otherwiſe deſcribe any Portion of that Circumference, to move towards the Center of the Circle over a Space or Diſtance of ſuch Length, as would ariſe from the Diviſion of the Square of that Portion of the Circumference by the Diameter of the Circle. And conſequently, it will in all Caſes be proportionable to the Square of the Arch the Body would deſcribe in a given Time divided by the Diameter of the Circle: Becauſe the Force exerted in any Time is proportionable to the Space it cauſes a Body to move over in that Time.

To illuſtrate this, ſuppoſe the Diameter of the Circle 25 Feet, and the Arch BN 10 Feet, and that the revolving Body moves over thoſe 10 Feet in a Minute. Then the Square of BN, which is 100, divided by the Diameter, or 25, gives 4 Feet: The central Force therefore at S, neceſſary to make the Body revolve in the Circle BNF at the Rate of 10 Feet in a Minute, muſt be ſuch, as ſuppoſing the Body left to itſelf at B, (that is, without any Motion at all towards C, or any Weight or Gravity of its own) would make it move over 4 Feet towards S in a Minute. Again, ſuppoſing Things as before, only that the Body revolves with but half its former Velocity, that is, that it deſcribes but 5 Feet of the Circumference of the Circle in a Minute; then the Square of that 5 Feet, which is 25, divided by 25, the Diameter gives 1; ſo that the central Force in this Caſe muſt be ſuch, that by Virtue thereof a Body deſtitute both of Weight and projectile Force, ſhall move over a Space 1 Foot long in a Minute. And converſely, if there be a Circle, whoſe Diameter is 25 Feet, and there be a Power lodged in the Center thereof capable of cauſing a Body to move over the Space of 4 Feet in a Minute; then in order that a Body may be made to revolve round in the Circumference of that Circle, it muſt be thrown forwards in a Direction at right Angles with the Radius of the Circle, with ſuch Velocity as would carry it over the Space of 10 Feet in that Time. And if the central Force be capable of cauſing it to move over but 1 Foot in a Minute, then it muſt be thrown forwards with a Velocity that ſhall make it move at the Rate of only 5 Feet: Or, ſince the Velocity is uniform, and therefore the Arch deſcribed is proportional to the Time it is deſcribed in, inſtead of an Arch as BN to calculate with, we may take the whole Circumference, and ſay, that *the Space a Body would move over by Virtue of the Central Force alone, in the Time it deſcribes the whole Circumference, is equal to that which would ariſe from the Diviſion of the Square of that whole Circumference by the Diameter of the Circle.* And this is that *exact* Adjuſtment between the central and projectile Force we have been ſo long in Search of.

Suppoſe then we would calculate, how far a Stone would fall by its own Weight, in the Time in which, if thrown forwards with a ſufficient Velocity, it would move quite round the Earth in a Circle. Let us call the Diameter of the Earth, that is, the Diameter of the Circle it deſcribes, 1, then the Circumference thereof would be 3 nearly, the Square of that 3, is 9, that divided by the Diameter 1, gives 9; ſo that a Stone would fall by Virtue of its own Weight through a Space equal to about 9 Diameters of the Earth in the Time it would revolve once round it. But it is here to be obſerved, that the Stone muſt be ſuppoſed to be of the ſame Weight during its whole Fall, that it is of at firſt; and ſo in all other Caſes, we ſuppoſe the Energy of the central Force to be the ſame at all Diſtances whatever, unleſs when the contrary is expreſſed.

We have hitherto been conſidering the Energy of the central Force, by comparing the Space it would cauſe a Body to move over in any certain Time, with the Space the Body ought to deſcribe along the Circumference of the Circle in the ſame Time: Let us now take it in another View, and compare the Velocity a Body ought to move with to deſcribe a Circle, with the Velocity it would acquire in moving over ſome certain Space by Virtue of the central Force only; from whence will ariſe a Propoſition very uſeful in Matters of this kind; and which for Diſtinction Sake we will call

### LEMMA II.

The Velocity a Body ought to revolve with in the Circumference of a Circle, is ſuch as it would acquire in moving by Virtue of the central Force only, over half the Radius of that Circle. To explain this, let the Body be ſuppoſed at Reſt at B, and let it be acted upon by a central Force at S. Upon this the Body will proceed towards S with a Motion, which will increaſe every Moment (becauſe as in the Caſe of falling Bodies we ſuppoſe the central Force acting upon it with the ſame Intenſeneſs, whether the Body moves or not) and the Inſtant it gets half Way to the Center S, it will have acquired ſome certain Degree of Velocity; that's the Velocity it ought to move with along the Circumference of the Circle to render its projectile Force an exact Counterbalance to the centripetal one of the Power at S. Thus, ſuppoſe there were a Loadſtone fixed at S, and a Piece of Steel at B, diveſted of its own Weight, ſo that it might be at Liberty to move freely towards the Loadſtone, and to make the Inſtance parallel to the Caſe before us, let it be ſuppoſed that the Loadſtone attracts the Steel with the ſame Degree of Force at all Diſtances, then whatever Velocity that Steel would have when it had got half way to the Loadſtone, that's a Velocity with which if the Steel were thrown from B along the Tangent BC, it would move round in the Path BDFB not approaching towards or receeding from the Loadſtone. But the Steel muſt be ſuppoſed to meet with no Reſiſtance from the Air, for if it does it will continually loſe ſomewhat of its Velocity, and ſo after ſome Revolutions would fall upon the Body of the Loadſtone. Again, ſuppoſe it were required to find the Velocity a Ball ought to be thrown forwards with at a ſmall Diſtance from the Surface of the Earth, that it ſhould move quite round in a Circle, keeping the ſame Diſtance from the Surface of it all the Way. It muſt be ſuch, as that Ball would acquire by falling half way from the Place it is thrown from to the Center of the Earth (for that is ſuppoſed to be the Seat of the Power that gives it its Weight, and thereby hinders it from moving forwards in the Direction it is thrown in.) And that Velocity is eaſily calculated from what was demonſtrated, Part I. Chap. V. § 4, or 5, ſuppoſing as *Hugens* has determined by very accurate Experiments, that a Body near the Surface of the Earth falls $15\frac{1}{12}$

*Paris* Feet in the firſt Second of Time. But enough of this by way of Explication, it is Time to demonſtrate what is here affirmed.

*Dem.* Let the Diameter of the Circle be 16 Feet, in which Caſe half the Radius will be 4 Feet, then by *Coroll.* of *Lemma* 1. the Arch the Body muſt run over in the Time it would paſs through thoſe 4 Feet by Virtue of the central Force, muſt be ſuch that the Square of it divided by the Diameter may be 4; that is, it muſt be 8 Feet, (becauſe the Square of 8, which is 64 divided by 16 the Diameter, gives 4,) but if it deſcribes an Arch of 8 Feet, in the Time it would fall through 4 Feet, it then moves with that very Velocity it might acquire by the Fall; becauſe as has been demonſtrated Part I. Chap. 5, § 7. if a Body moves uniformly with the Velocity it might acquire by a Fall, it will move over a Space equal to *twice* the Height of the Fall, in the Time thereof. Since then the Body in the Caſe before us, deſcribes an Arch of the Circumference of 8 Feet in the Time it would fall through 4 Feet, it moves therein with a *Velocity* it would acquire by falling through 4 Feet, that is, half the Radius. Q. E. D. To avoid a Circumlocution in the midſt of the Argument, I call the Space the Body would move over by Virtue of the central Force only, from its Analogy therewith, a *Fall*.

*Univerſally thus*, let the Diameter be equal to 4 times ſome certain Space, which Space let us call *d*; the Diameter then will be equal to 4 *d*, and half the Radius will be equal to the Space *d*, in which Caſe, the Arch the Body deſcribes in the Time it would fall through the Space *d* muſt by the abovementioned Corollary, be ſuch as that the Square of it divided by 4 times *d* may be once *d*, that is, the Arch muſt be twice *d* (becauſe the Square of twice *d*, which is 4 times *dd* divided by the Diameter 4 times *d*, gives once *d*) but as before, if the Body move uniformly through *twice* the Space *d*, in the time it would fall through *once* that Space, it then moves with that very Velocity it would acquire by falling through that Space, that is, *ex Hypoth.* through half the Length of the Radius. Q. E. D.

From what has been demonſtrated above, we may eaſily learn the Relation which the central Force that is requiſite to retain one Body in the Circle it deſcribes, has to that which is required for the ſame, or another Body revolving in a larger or leſs Circle. Which Relation is expreſſed in the following *Lemma*.

## LEMMA III.

The central Force whereby one Body is retained in the Circle it deſcribes, is to that whereby a Body moving with a different Velocity or in a different Circle might be retained herein, as the Square of the Velocity of the former divided by the Radius of the Circle it deſcribes, to the Square of the Velocity of the latter divided by the Radius of the Circle which it deſcribes. If then we call the central Forces whereby the two revolving Bodies are retain'd in their reſpective Circles C and *c*, their Velocities V and *v*, and the Diſtances they revolve at, that is, the Radii of the Circles they deſcribe, R and *r*, we ſhall always have this Proportion, $C : c :: \frac{VV}{R} : \frac{vv}{r}$.

*Dem.* By Corollary of Lemma 1, the Energy of the central Force is proportionable to the Square of the Arch the Body deſcribes in a given Time, divided by the Diameter of the Circle. And Arches deſcribed in a given Time, are as the Velocities whereby they are deſcribed, and the Diameter of a Circle is as its Radius; conſequently the Action of the central Force is proportionable to the Square of the Velocity the Body moves with divided by the Radius of the Circle it deſcribes: The central Force therefore whereby one Body is retained in the Circle it deſcribes, is to that whereby a Body moving with a different Velocity or in a different Circle might be retained therein, as the Square of the Velocity of the former divided by the Radius of the Circle it deſcribes, to the Square of the Velocity of the latter, divided by the Radius of the Circle which it deſcribes. Q. E. D.

Theſe Things which are the Foundation of what follows being eſtabliſhed, we may now proceed to demonſtrate each Propoſition of this Chapter in the Order they lie in; the firſt of which is, That when two or more Bodies revolve at *equal* Diſtances from the Center, but with *unequal* Velocities, the central Forces neceſſary to retain them will be to each other, as the Squares of their Velocities.

*Dem.* Let the Circle which one of the revolving Bodies is ſuppoſed to deſcribe in this and the following Caſes, be BDF, in Fig. 26. and that which the other deſcribes be *b d f*, and let us call the Diſtances they revolve at from the common Center, that is, the Radii of the Circles they deſcribe, R and *r*; the Velocities they revolve with, V and *v*; and the central Forces neceſſary to retain them C and *c*. Then by Lemma 3. we ſhall

have for the firſt ſtep of this Demonſtration, 1 $C : c :: \frac{VV}{R} : \frac{vv}{r}$

But by the Propoſition R is equal to *r*, therefore 2 $C : c :: VV : vv$. Q. E. D.

**PROP. II.** When two or more Bodies move with *equal* Velocities, but at *unequal* Diſtances from the Center they revolve about, their central Forces muſt be inverſely as their Diſtances. That is, by how many times greater the Diſtance a Body revolves at is from the Center, ſo many times leſs Force will retain it (*b*).

**PROP. III.** When two or more Bodies perform their Revolutions in *equal* Times, but at *different* Diſtances from the Center they revolve about; the Forces requiſite to retain them in their Orbs will be to each other as the Diſtances they revolve at from the Center. For Inſtance, if one revolves at twice the Diſtance the other does, it will require a double Force to retain it; if it deſcribes a Circle three times as large as the other does, then a treble Force is requiſite, &c. (*c*)

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(*b*) *Dem.* By Lemma 3, as before, 1 $C : c :: \frac{VV}{R} : \frac{vv}{r}$

By the Propoſition V is equal to $v$, therefore, 2 $C : c :: \frac{1}{R} : \frac{1}{r}$. Q. E. D.

(*c*) In order to demonſtrate this the following Lemma will be of uſe.

### LEMMA IV.

The periodical Time of a Body revolving in a Circle, is as the Diſtance it revolves at from the Center *directly*, and as its Velocity *inverſly*. And therefore in comparing the periodical Times of two Bodies together, if we call their periodical Times T and $t$, the Diſtances they revolve at, R and $r$; and their Velocities V and $v$, as before, we ſhall have this Propor-

**Prop. IV.** When two or more Bodies revolving at *different* Diſtances from the Center, are retained by *equal* centripetal Forces, their Velocities will be ſuch, that their periodical Times will be to each other as the Square Roots of their Diſtances. That is, if one revolves at four times the Diſtance another does, it will perform a Revolution in twice the Time the other does; if at nine times the Diſtance, it will revolve in thrice the Time, &c. *(d)*

Proportion, *viz.* $T : t :: \frac{R}{V} : \frac{r}{v}$.

*Demonſtration of the Lemma.* The periodical Time of a revolving Body, *cæteris paribus*, depends on the Diſtance it revolves at from the Center, becauſe the greater that Diſtance is, the greater is the Circle it deſcribes, and ſo the longer it is in coming round. Again, *cæteris paribus*, the greater its Velocity is, the ſooner it comes round; and therefore the periodical Time is directly as the former, and inverſely as the latter: Or, as the former divided by the latter; and conſequently $T : t :: \frac{R}{V} : \frac{r}{v}$ Q. E. D.

| | | |
|---|---|---|
| *Demonſt. of the Propoſition.* By Lemma 3. | 1 | $C : c :: \frac{VV}{R} : \frac{vv}{r}$ |
| By Lemma 4, | 2 | $T : t :: \frac{R}{V} : \frac{r}{v}$ |
| Multiplying the 2d Step by the Ratio of V to *v*, we have, | 3 | $TV : tv :: R : r$ |
| By the Propoſition, T is equal to *t*, therefore, | 4 | $V : v :: R : r$ |
| Comparing the 1ſt and 4th Steps together, | 5 | $C : c :: \frac{VV}{V} : \frac{vv}{v}$ |
| That is, | 6 | $C : c :: V : v$. Q. E. D. |

| | | |
|---|---|---|
| (*d*) *Dem.* By Lemma 3. | 1 | $C : c :: \frac{VV}{R} : \frac{vv}{r}$ |
| Multiplying by $R : r$ | 2 | $CR : cr :: VV : vv$ |
| By the Propoſition, C is equal to *c*, therefore | 3 | $R : r :: VV \quad vv$ |

By

**Prop. V.** And in general, whatever be the Diſtances, the Velocities, or the periodical Times of the revolving Bodies, the retaining Forces will be to each other in a Ratio compounded of their Diſtances directly, and the Squares of their periodical Times inverſely. Thus for Inſtance, if one revolves at twice the Diſtance another does, and is three times as long in moving round, it will require $\frac{2}{9}$, that is, *two ninths* of the retaining Power the other does. For as, if the retaining Power was in a Ratio compounded of the Diſtance *directly* and *directly* alſo of the Square of the periodical Time, the latter Body would require *twice* the retaining Power the other would *multiplied by* 9, the Square of the periodical Time; ſo, as the latter Ratio is an *inverſe* one, it requires twice that Power *divided by* 9; that is, $\frac{2}{9}$ or *two ninths* of it. For in compounding of Ratio's the Way is, if they are both *direct* to *multiply* them both together; if one be *direct* and the other *inverſe*, to divide the *direct* by the *inverſe*; if both are *inverſe*, to multiply them both together, and to divide *Unity* by them: Thus, if the retaining Power had been

| | | |
|---|---|---|
| By Lemma 4. | 4 | $T : t :: \frac{R}{V} : \frac{r}{v}$ |
| Squaring the laſt Step | 5 | $TT : tt :: \frac{RR}{VV} : \frac{rr}{vv}$ |
| Comparing the 3d and 5th, | 6 | $TT : tt :: \frac{RR}{R} : \frac{rr}{r}$ |
| That is, | 7 | $TT : tt :: R : r$ |
| Extracting the Roots of the 7th Step, | 8 | $T : t :: \sqrt{R} : \sqrt{r}$ Q.E.D. |

as the Diſtance *inverſely* and *inverſely* too as the Square of the periodical Time, then in the Caſe before us, the latter Body would have required a retaining Power, which ſhould have been as the Number 1 divided by 2, multiplied by 9; that is as 1 divided by 18: That is, if we ſuppoſe 1 to expreſs the central Force requiſite for the former Body, the Fraction $\frac{1}{18}$ would have expreſſed that which would have been neceſſary for the latter; or in other Words, their retaining Powers would have been related to each other, as 1 is to $\frac{1}{18}$; that is, the former would have been 18 times greater than the latter (*e*).

**Prop. VI.** If ſeveral Bodies revolve at different Diſtances from one common Center, and the retaining Power lodged in that Center decreaſes as the Squares of the Diſtances increaſe; that is, if at a double Diſtance it be

| | | |
|---|---|---|
| (*e*) *Dem.* By Lemma 3. | 1 | $C : c :: \frac{VV}{R} : \frac{vv}{r}$ |
| Multiplying by R to *r*, | 2 | $CR : cr :: VV : vv$ |
| By Lemma 4, | 3 | $T : t :: \frac{R}{V} : \frac{r}{v}$ |
| Multiplying the laſt by V to *v*, | 4 | $TV : tv :: R : r$ |
| Dividing the laſt by T to *t*, | 5 | $V : v :: \frac{R}{T} : \frac{r}{t}$ |
| Squaring the laſt, | 6 | $VV : vv :: \frac{RR}{TT} : \frac{rr}{tt}$ |
| Comparing the 2d and 6th, | 7 | $CR : cr :: \frac{RR}{TT} : \frac{rr}{tt}$ |
| Dividing the laſt by R to *r* | 8 | $C : c :: \frac{R}{TT} : \frac{r}{tt}$. Q. E. D. |

four times weaker; and at a treble Diſtance, 9 times weaker, *&c.* the Squares of the periodical Times of thoſe Bodies will be to each other as the Cubes of their Diſtances from the common Center. That is, if there be two Bodies, whoſe Diſtances when *cubed*, that is, multiplied by themſelves twice, are *double* or *treble*, *&c.* of each other, then their periodical Times will be ſuch, as that when *ſquared* only, that is, multiplied by themſelves *once*, they ſhall be alſo *double*, or *treble*, *&c.* of each other (*f*).

| | | |
|---|---|---|
| (*f*) *Dem.* By Lemma 3, | 1 | $C : c :: \frac{VV}{R} : \frac{vv}{r}$ |
| By the Propoſition, | 2 | $C : c :: \frac{1}{RR} : \frac{1}{rr}$ |
| Comparing the 1ſt and 2d Step, | 3 | $\frac{VV}{R} : \frac{vv}{r} :: \frac{1}{RR} : \frac{1}{rr}$ |
| Multiplying by R to *r*, | 4 | $VV : vv :: \frac{1}{R} : \frac{1}{r}$ |
| By Lemma 4, | 5 | $T : t :: \frac{R}{V} : \frac{r}{v}$ |
| Multiplying by V to *v*, | 6 | $TV : tv :: R : r$ |
| Dividing by T to *t*, | 7 | $V : v :: \frac{R}{T} : \frac{r}{t}$ |
| Squaring the laſt, | 8 | $VV : vv :: \frac{RR}{TT} : \frac{rr}{tt}$ |
| Comparing the 4th and the 8th, | 9 | $\frac{1}{R} : \frac{1}{r} :: \frac{RR}{TT} : \frac{rr}{tt}$ |
| Multiplying by TT to *tt* | 10 | $\frac{TT}{R} : \frac{tt}{r} :: RR : rr$ |
| Multiplying by R to *r*. | 11 | $TT : tt :: RRR : rrr$. Q.E.D. |

And after like manner, we may eaſily demonſtrate the ſeventh Corollary of the fourth Propoſition of the firſt Book of Sir *Iſaac Newton*'s *Principia*. Which becauſe it is of more than ordinary Difficulty to Beginners, I here add. It is to this Effect.

When Bodies revolve in ſuch Manner that their periodical Times are to each other as ſome Power (which call *n*) of their

Diſtances, the centripetal Forces requiſite to retain them, will be to each other inverſely as twice that Power wanting one of their Diſtances; that is, as their Diſtances raiſed to the Power $2n-1$.

| | | |
|---|---|---|
| *Dem.* By Lemma 3, | 1 | $C : c :: \frac{VV}{R} : \frac{vv}{r}$ |
| Multiplying by the Ratio of R to $r$, | 2 | $CR : cr :: VV : vv$ |
| By the Suppoſition, | 3 | $T : t :: R^n : r^n$ |
| By Lemma 4, | 4 | $T : t :: \frac{R}{V} : \frac{r}{v}$ |
| Comparing the 3d and 4th, | 5 | $R^n : r^n :: \frac{R}{V} : \frac{r}{v}$ |
| Multiplying by V to $v$, | 6 | $VR^n : vr^n :: R : r$ |
| Dividing by $R^n$ to $r^n$, | 7 | $V : v :: \frac{R}{R^n} : \frac{r}{r^n}$ |
| Dividing the Numerators in the laſt Step by $R : r$, gives $1 : 1$, and dividing the Denominators $R^n : r^n$ by the ſame, brings them one Power lower, that is, to $R^{n-1} : r^{n-1}$ therefore the laſt Step is reducible to | 8 | $V : v :: \frac{1}{R^{n-1}} : \frac{1}{r^{n-1}}$ |
| Squaring both Sides of the 8th Step, which in the Denominators $R^{n-1}$ and $r^{n-1}$ is done by doubling their Index $n-1$, we have | 9 | $VV : vv :: \frac{1}{R^{2n-2}} : \frac{1}{r^{2n-2}}$ |
| Comparing the 2d and 9th Steps, | 10 | $CR : cr :: \frac{1}{R^{2n-2}} : \frac{1}{r^{2n-2}}$ |
| Dividing by R to $r$, which in the Denominators $R^{2n-2}$ & $r^{2n-2}$ is done by adding 1 to their Index $2n-2$, by which means it becomes $2n-1$, we have | 11 | $C : c :: \frac{1}{R^{2n-1}} : \frac{1}{r^{2n-1}}$<br>Q. E. D. |

*Scholium.* The foregoing Demonſtrations would have proceeded with equal Succeſs, had we only taken the Antecedents, or the Conſequents in each Step; that is, only the large or only the ſmall Letters. Which Method, becauſe I ſhall make uſe of it hereafter, I will here give a Specimen of. Let it then be required to demonſtrate the firſt Propoſition of this Chapter over again in this latter Method.

| | | |
|---|---|---|
| By Lemma 3. The central Forces are as the Squares of the Velocities divided by the Radii of the Circles deſcribed, which put thus | 1 | $C : \frac{VV}{R}$ |
| By the Propoſition, the Circles deſcribed have one common Radius, call it 1. Then | 2 | $R = 1$ |
| From the 1ſt and 2d Step compared, we have | 3 | $C : VV$. |

That is, the central Forces of the revolving Bodies are to each other as the Squares of their Velocities, which is the Senſe of the Propoſition.

Again, let it be required to demonſtrate the Corollary taken from Sir *Iſaac Newton*'s *Principia*, over again in this Method.

| | | |
|---|---|---|
| By Lemma 3. —— | 1 | $C : \frac{VV}{R}$ |
| By the Suppoſition —— | 2 | $T : R^n$ |
| By Lemma 4. —— | 3 | $T : \frac{R}{V}$ |
| Comparing the 2d and 3d Steps | 4 | $\frac{R}{V} : R^n$ |
| Multiplying by V | 5 | $R : VR^n$ |
| Dividing both Sides by R, reduces R to 1, and brings $R^n$ one Power lower, that is to $R^{n-1}$, ſo that we ſhall then have | 6 | $1 : VR^{n-1}$ |
| Dividing by $R^{n-1}$ | 7 | $\frac{1}{R^{n-1}} : V$ |
| Squaring the laſt Step, which in the Quantity $R^{n-1}$ is effected by doubling the Index $n-1$, we have | 8 | $\frac{1}{R^{2n-2}} : VV$ |
| Multiplying the firſt Step by R | 9 | $CR : VV$ |
| Comparing the two laſt | 10 | $CR : \frac{1}{R^{2n-2}}$ |
| Dividing by R, which in the Quantity $R^{2n-2}$ is done by adding 1 to the Index, we have | 11 | $C : \frac{1}{R^{2n-1}}$. |

Q. E. D.

## *Secondly, Of the* Forces *neceſſary to retain Bodies revolving in* other Orbs.

**Prop. VII.** If a Body be turned out of its rectilineal Courſe by Virtue of a central Force, which decreaſes as you go from the Seat thereof, as the Squares of the Diſtances increaſe; that is, which is inverſely as the Square of the Diſtance, the Figure that Body ſhall deſcribe, if not a Circle, will be a Parabola, an Ellipſe or an Hyperbola, and one of the Foci of the Figure will be at the Seat of the retaining Power. That is, if there be not that exact Adjuſtment between the projectile Force of the Body and the central Power neceſſary to cauſe it to deſcribe a Circle, it will then deſcribe one of thoſe other Figures, one of whoſe Foci will be where the Seat of the retaining Power is *(g)*.

*(g)* In order to ſhew this, it will be neceſſary to premiſe the following Lemma's.

LEMMA V.

The Velocity of a Body deſcribing any Curve, (if its Courſe is regulated by a retaining Power) is in one Point of the Curve to its Velocity in any other Point thereof, inverſely as a Perpendicular drawn from the Seat of the retaining Power to a Tangent of the Curve at the firſt Point, to a Perpendicular drawn from the ſame Place to a Tangent at the other Point. That is, in fewer Words, the Velocity of the Body is inverſely as a Perpendicular let fall from the Seat of the retaining Power to a Tangent at the Point where the Body is. Thus, if the Body be at D, a Point in Curve DEF, Fig. 27, and PD be a Tangent thereto, and SP a Perpendicular to that Tangent, drawn through S the Seat of the retaining Power, the Velocity of the Body at D, compared with its Velocity in other Parts of the Figure, will be inverſely as SP, that is, as the Quantity $\frac{1}{SP}$.

*Dem.* It has been demonſtrated, Part I. Chap. VIII. that revolving Bodies deſcribe equal Areas in equal Times; that is, if a Body deſcribes the Curve Line DEF, and the Arches DE, EF, *&c.* be run over in equal Times, the Areas DES, EFS, *&c.* will be equal; which Areas if we ſuppoſe the Times as ſmall as poſſible, may be conſidered as ſo many right-lined Triangles, becauſe in that Caſe the Lines DE, EF, *&c.* looſe their Curvity, and being produced are Tangents to the Curve. Produce the Line DE to P, and let fall the Perpendicular SP, then the Area of the Triangle DES is had by multiplying its Baſe DE into half SP; and the Area of EFS is equal to its Baſe EF multiplied by half a Perpendicular drawn from S to EF produced, *&c.* But theſe Triangles being deſcribed in equal Times, are all equal, the longer therefore their Baſes, the ſhorter their Perpendiculars. But the Baſes being run over in equal Times by the revolving Body, they are as the Velocity of the Body wherewith they are deſcribed; conſequently the greater the Velocity of the Body, the ſhorter is the Perpendicular to the Tangent where the Body is. Which is the Senſe of the Lemma.

## LEMMA VI.

Whatever Curve a revolving Body deſcribes by Virtue of a centripetal Force, whether the Seat of that Force be within or without the Curve, the Action of that Force upon the Body when at any one Point of the Curve, muſt be to the ſame upon the Body when at another Point of the Curve, directly as the Diſtance of the Body from the Seat of the retaining Power and inverſely as the Cube of a Perpendicular let fall from the Seat of the retaining Power to a Tangent to the Curve at the Point where the Body is, multiplied by the Radius of a Circle of equal Curvature with the Figure at that Point. That is, if AX in Fig. 28, be the Curve deſcribed, and MDN, whoſe Center is C, be a Circle of equal Curvature with the Figure at the Point D, and PD be a Tangent to that Point, and SP a Perpendicular thereto drawn thro' S the Seat of the retaining Power, the central Force requiſite to act upon the Body at D muſt be as $\frac{DS}{SP^3 \times CD}$, that is, if we put R for the Radius CD, as $\frac{DS}{\overline{SP^3} \times \overline{R}}$.

*Dem.* By Lemma 5. the Velocity of the revolving Body when at D is as $\frac{1}{SP}$, calling then that Velocity V, we have —— 1 $V : \frac{1}{SP}$

Squaring both Sides — 2 $VV : \frac{1}{SPq}$

Call CD the Radius of the Circle MDN, R; then by Lemma 3, the central Force neceſſary to retain the Body in that Circle, (or which is the ſame thing, at the Point D in the Curve ADX, becauſe they are both of equal Curvature there) would, if placed at C, be as $\frac{VV}{R}$. But whereas it is placed at S it acts obliquely to DC, and therefore to produce the ſame Effect, muſt act more forcibly in the Proportion of SD to DT, ſuppoſing ST perpendicular to DC; that is, becauſe SP is equal to DT, in the Proportion of SD to SP.

But by the Golden Rule 3 $SP : SD :: \frac{VV}{R} : \frac{VV \times SD}{SP \times R}$

Conſequently the central Force neceſſary if placed not at C but at S, is expreſſible by 4 $\frac{VV \times SD}{SP \times R}$

Putting then C for the Action of the central Force neceſſary to be placed at S, we have 5 $C : \frac{VV \times SD}{SP \times R}$ or $VV \times \frac{SD}{SP \times R}$

Comparing the 2d and 4th Steps together 6 $C : \frac{1}{SPq} \times \frac{SD}{SP \times R}$

That is, 7 $C : \frac{SD}{SP3 \times R}$. Q. E. D.

## LEMMA VII.

Let AD Fig. 29 be a Parabola, whoſe Axis is AG, and its Focus S, and let PDF be a Tangent at the Point D, through the Focus S and Point of Contact D draw the Lines PS and DG both perpendicular to the Tangent PD, then will PS be equal to half DG.

*Dem.* Produce DP till it meets the Axis in X, and draw the Diameter DO, and join the Points D and S: Then becauſe DO, as being a Diameter is parallel to G X, the Angle FDO is equal to DXS; 'tis alſo equal to the Angle XDS (Miln. Conic. Sect. Part IV. Prop. 4.) the Triangle XSD is therefore an Iſoſceles one, and SP being perpendicular to the Baſe XD, XP is equal to half XD; but PS and DG being parallel, the Triangles XPS and

XDG are ſimilar, and therefore PS is alſo equal to half DG. Q. E. D.

## LEMMA VIII.

Let AD Fig. 30, be a Portion of a Parabola, an Ellipſe or an Hyperbola, one of whoſe Foci is S, its longer Axis AG, and its Parameter or Latus rectum L; and let PF be a Tangent at the Point D, join the Points D and S, through the latter of which draw the Line SP perpendicular to the Tangent, and through the former the Line DG perpendicular alſo to the Tangent, and croſſing the longer Axis in G, then will DG be equal to $\frac{L\times SD}{2SP}$.

*Dem.* Through the Point G draw the Line GT perpendicular to DS, then will the right-angled Triangles DGT and DSP be ſimilar, becauſe the Angles DSP and SDG are alternate, conſequently

| | | |
|---|---|---|
| | 1 | $DG : DT :: SD : SP$ |
| But by Miln. Conic. Sect. Part IV. Prop. 6. | 2 | $DT = \frac{1}{2} L$ |
| From the firſt and 2d Step compared together | 3 | $DG : \frac{1}{2} L :: SD : SP$ |
| Turning the 3d Step into an Equation. | 4 | $DG = \frac{L\times SD}{2SP}$. Q. E. D. |

## LEMMA IX.

Things remaining as before, produce DG to C in Fig. 31, 32, 33, ſo that the Line DC may be equal to $\frac{L\times SD^3}{2SP^3}$, then will a Circle whoſe Center is C, and Radius CD, be of the ſame Curvature, and coincident with the Figure AD at the Point D, whether it be a Parabola, an Ellipſe or an Hyperbola.

### *Demonſtration.*

I. *For the Parabola.* The ſame Lines being drawn as in the foregoing Figure, draw alſo through D, Fig. 31. the Diameter DY, and take a Portion of it as DI equal in Length to the Parameter of that Diameter, and let fall the Perpendicular IH meeting DG produced in H, and let L be the Latus rectum or Parameter of the Figure, then by the 3d Step of Lemma 8.

| | | |
|---|---|---|
| | 1 | $DG : \frac{1}{2} L :: SD : SP$ |
| The Angle HDI wants IDF to make it a right one, the Angle PSD wants SDP to make it a right one, but IDF and SDP are equal, Miln. Conic. Sect. Part IV. Prop. 4. therefore HDI and PSD are ſo alſo, the Triangles therefore HDI and SDP are ſimilar; conſequently | 2 | $SD : SP :: DH : DI$. |

From

| | | |
|---|---|---|
| From the 1ſt and 2d Steps compared together | 3 | $DG : \frac{1}{2} L :: DH : DI$ |
| By Miln. Conic. Sect. Part IV. Prop. 10. Corol. | 4 | $DI = 4\,DS$ |
| From the 3d and 4th Step compared | 5 | $DG : \frac{1}{2} L :: DH : 4\,DS$ |
| Multiplying the Antecedents of the firſt Step by 4. | 6 | $4\,DG : \frac{1}{2} L :: 4\,SD : SP$ |
| Compounding the fifth and ſixth Steps, that is multiplying them together | 7 | $4\,DG^2 : \frac{LL}{4} :: DH : SP$ |
| Lemma 7. | 8 | $\frac{1}{2} DG = SP$ |
| Comparing the 7th and 8th Steps | 9 | $4\,DG^2 : \frac{LL}{4} :: DH : \frac{1}{2} DG$ |
| Turning the 9th Step into an Equation | 10 | $2\,DG^3 = \frac{LL \times DH}{4}$ |
| Taking half the 10th Step | 11 | $DG^3 = \frac{LL \times DH}{8}$ |
| From Lemma 8, | 12 | $DG^3 = \frac{L^3 \times SD^3}{8SP^3}$ |
| Comparing the 11th and 12th Steps | 13 | $\frac{DH \times LL}{8} = \frac{LLL \times SD^3}{8SP^3}$ |
| Dividing the 13th Step by $\frac{LL}{8}$ | 14 | $DH = \frac{L \times SD^3}{SP^3}$ |
| Halving the 14th Step | 15 | $\frac{1}{2} DH = \frac{L \times SD^3}{2SP^3}$ |
| But by Suppoſition | 16 | $CD = \frac{L \times SD^3}{2SP^3}$ |
| Comparing the two laſt Steps | 17 | $CD = \frac{1}{2} DH$ |

But DH being perpendicular to PF, and the Angle at I a right one, it is obvious a Circle whoſe Diameter is DH, (and conſequently by the laſt Step, whoſe Radius is DC,) will paſs through the Point I, 31. El. 3. and will have the Line PF for a Tangent to it at D; but by Miln. Conic. Sect. Part III. Prop. 11. Corol. 1. ſuch a Circle will be of equal Curvature with the Parabola at the Point D; and the Circle and the Parabola will be alſo coincident with each other at that Point, becauſe the Line PF is a Tangent common to both. Q. E. D.

II. *For the Ellipſis and Hyperbola:* Let AD in Fig. 32, be a Portion of an Ellipſis, in Fig. 33, a Portion of an Hyperbola, one of whoſe Foci is S, its Latus Rectum L, and its longer Axis AB; let PF be a Tangent at the Point D, and draw the Lines SP, SD, and DG as in Fig. 30; draw alſo the Diameter D*d* and produce it to I, ſo that DI may be equal to the Parameter of that Diameter; let fall the Perpendicular IH meeting DG produced in H; through the Center E draw the Semiaxis EQ, and the Diameter K*k* conjugate to D*d*; and to the longer Axis AB, the Ordinates DM and KN; and in the Hyperbola produce EK and HD till they meet in R. Then by the known Property of the Ellipſis and Hyberbola De L'Hoſpit. Liv. 2. Prop. 2. and

| | | |
|---|---|---|
| Liv. 3. Prop. 2. | 1 | $EQ^q \; CA^q :: DM^q : AM \times MB$ |
| Simſon's Conic. Sect. Lib. 2. Prop. 19. and Lib. 3. Prop. 43. | 2 | $AM \times MB = EN^q$ |
| Comparing the 1ſt and 2d Steps together, and extracting their Roots | 3 | $EQ : EA :: DM : EN$ |

The Diameter K*k* being conjugate to D*d* is by the Definition of ſuch a Diameter, parallel to the Tangent PF, and conſequently DH being perpendicular to that Tangent, the Triangle GER is a right-angled one, that Triangle therefore and the right-angled one KEN are ſimilar as having in the Ellipſe, their Angles at E vertical; in the Hyperbola, common: The rightangled Triangles GER and DGM are alſo ſimilar, as having their Angles at G in the Ellipſe, vertical; in the Hyperbola, common: And conſequently the Triangles KEN and DGM are ſimilar in each Figure.

| | | |
|---|---|---|
| Therefore | 4 | $DM : EN :: DG : EK$ |
| Comparing the 3d and 4th Steps together | 5 | $EQ : EA :: DG : EK.$ |
| The rightangled Triangles DER and DHI in the Ellipſe are ſimilar as having their Angles at D common, in the Hyperbola as having their Angles at D vertical, conſequently | 6 | $DE : DR :: DH : DI.$ |

| | | |
|---|---|---|
| By the Definition of the Parameter DI, for the Ellipſe De L'Hoſpit. Lib. 2. Definit. 13. for the Hyperbola Lib. 3. Definit. 15. | 7 | $DE : EK :: 2\,EK : DI$ |
| The Extremes of the two laſt Steps being the ſame, the Product of the Means of the one muſt be equal to the Product of the Means of the other, that is | 8 | $2\,EK^q = DH \times DR$ |
| Multiplying the 8th Step by EK | 9 | $2\,EK^3 = DH \times DR \times EK$ |
| By Simſon's Con. Sect. Lib. 2. Prop. 20. for the Ellipſe, Lib. 3. Prop. 45. for the Hyperbola | 10 | $DR \times EK = EA \times EQ$ |
| From the 9 and 10. | 11 | $2\,EK^3 = DH \times EA \times EQ$ |
| Dividing the 11 by DH | 12 | $\frac{2\,EK^3}{DH} = EA \times EQ$ |
| By the Definit. of a principal Parameter | 13 | $\frac{1}{2}L : EQ :: EQ : EA$ |
| Wherefore from the laſt by Definit. 10. Elem. 5. | 14 | $\frac{1}{2}L : EA :: EQ^q : EA^q$ |
| Multiplying the two laſt together | 15 | $\frac{LL}{4} : EQ \times EA :: EQ^3 : EA^3$ |
| Comparing the 5th and 15th together, we have | 16 | $\frac{LL}{4} : EQ \times EA :: DG^3 : EK^3$ |
| Multiplying the two laſt Terms of the 16th Step by $\frac{2}{DH}$, it becomes | 17 | $\frac{LL}{4} : EQ \times EA :: \frac{2\,DG^3}{DH} : \frac{2\,EK^3}{DH}$ |

| | | |
|---|---|---|
| Comparing the 12th Step and the 17th together | 18 | $\frac{LL}{4} : EQ \times EA :: \frac{2DG^3}{DH} : EA \times EQ$ |
| The Conſequents in the 18th Step being the ſame, the Antecedents are equal, that is | 19 | $\frac{2DG^3}{DH} = \frac{LL}{4}$ |
| Multiplying by DH | 20 | $2DG^3 = \frac{LL \times DH}{4}.$ |

But the laſt Step here is the ſame with the 10th in the Demonſtration for the Parabola, from whence therefore the ſame Concluſion is to be drawn here as there: for the Propoſition in Miln. Conic. Sect. referred to at the End of that Demonſtration, relates alſo to both the Ellipſe and the Hyperbola.

Demonſtration of the Propoſition above, to which this Note refers.

Let ADX, Fig. 28. be a Portion of the Figure the revolving Body deſcribes, whether it be a *Parabola*, an *Ellipſe*, or an *Hyperbola*; and let L be the Latus Rectum or Parameter of the Figure; let the Seat of the retaining Power be at S, one of the Foci thereof, and let the revolving Body be ſuppoſed at D: draw the Line PD a Tangent to the Curve at that Point, and let fall the Perpendicular SP. Perpendicular to the Tangent draw the Line DC, which let us ſuppoſe to be the Radius of a Circle as MDN, of the ſame Curvature, that the Figure ADX is at the Point D. Then it is obvious, that the ſame Force which placed at S would retain the revolving Body in the Circumference of the Circle when it came at D, will retain it in the other Figure, at that Point, they being at that Place one and the ſame Curve. But by Lemma 6, ſuch Force is as $\frac{DS}{SP^3 \times R}$. Calling then the retaining Power C; we ſhall have this Proportion, *viz.*

| | | |
|---|---|---|
| | 1 | $C : \frac{DS}{SP^3 \times R}$ |

And ſince the Circle MDN is of the ſame Curvature and coincident with the Figure ADX, its Radius DC is by Lemma 9, equal to the Quantity $\frac{L \times SD^3}{2\ SP^3}$, putting then R for the Radius as

| | | |
|---|---|---|
| before we ſhall have this Equation | 2 | $R = \frac{L \times SD^3}{2\ SP^3}$ |
| Comparing the 1ſt and 2d Step together, we ſhall have this Proportion | 3 | $C : \frac{DS}{SP^3 \times \frac{L \times SD^3}{2\ SP^3}}$ |
| Expunging SP³ as being both Multiplicator and Diviſor | 4 | $C : \frac{DS}{\frac{1}{2} L \times SD^3}$ |

Dividing

That a Body ſhall deſcribe Figures ſo different, when the Power that directs its Courſe is the ſame, and acts by one and the ſame Law, is owing to the Degree of Velocity the Body moves with. Thus ſuppoſing the central Power placed at S (in Fig. 34.) and that the revolving Body ſets off at D in the Direction DB perpendicular to the Line DS, with ſuch Velocity as it would obtain in falling by Virtue of that central Power only, half way from D to S, it will then, by Lemma 2, deſcribe a Circle as DKM, whoſe Center is the Point S. If it ſets off from D in the ſame Direction as before, but with a leſs Velocity, it then revolves within that Circle deſcribing an Ellipſe, as DLN, one of whoſe *Foci* is at S, the Seat of the retaining Power, and the other between that and the Point D, as at F. If it ſets off from D with ſomewhat greater Velocity than what it would acquire by the above-mentioned Fall, it will ſtill deſcribe an Ellipſe, as DEO, one of whoſe *Foci* ſhall be at S as before, but

| | | |
|---|---|---|
| Dividing all by DS | 5 | $C : \frac{1}{\frac{1}{2} L \times DS^q}$ |
| But $\frac{1}{2}$ L is the ſame, whatever Point of the Figure deſcribed the revolving Body is at, and may therefore be conſidered as Unity, conſequently | 6 | $C : \frac{1}{DS^q}$ |

That is, the retaining Power, if it be inverſely as the Square of the Diſtance, and ſeated in the *Focus* of the Figure, whether it be a *Parabola*, an *Ellipſe* or an *Hyperbola*, will direct the Courſe of the revolving Body in ſuch Manner, as that it ſhall deſcribe the ſaid Figure. Q. E. D.

the other ſhall be beyond it as at G. If it ſets off with a Velocity which is greater than that it would acquire by the above-mentioned Fall, in the Proportion of $\sqrt{2}$ to 1, that is, the Square Root of 2 to 1, it ſtill deſcribes a Figure, one of whoſe *Foci* is S as before, but the other, as G, goes off to an infinite Diſtance; that is, the Figure deſcribed becomes a Parabola, as DPQ, whoſe Focus is S. If it ſets off with a greater Velocity than this laſt, the Seat of the retaining Power remains ſtill at S, but the other *Focus* G goes yet farther off; that is, it comes on on the other Side the Point D as at H, and the Figure deſcribed becomes an Hyperbola, as DRT, whoſe Foci are S and H.

From hence it is obſervable, that of the four Figures there are but two wherein an exact Adjuſtment is requiſite between the retaining Power and the projectile Force, *viz.* the Circle and the Parabola. In the former of which, the Velocity muſt be ſuch as the Body would acquire by falling half way to S, in the latter it muſt exceed that in the Proportion of the Square Root of 2 to 1; that is, in the Proportion of ſomewhat leſs than 1½ to 1. Thus for Inſtance, if one Degree of Velocity is requiſite to cauſe a Body to revolve in a Circle about the Point S; it will require about one Degree and an half to make it deſcribe a Parabola by Virtue of the ſame retaining Power ſeated in the ſame Point S.

Again, the ſame Figures may be deſcribed, though the Seat of the retaining Power be in the other Foci, F, G, or H; for Inſtance, if it be at F, in order that the Body may deſcribe the Ellipſe DLN, it muſt ſet off at D with the ſame Velocity it had in the former Caſe when it came to N: If it be at G, it will deſcribe the Ellipſe DEO, ſetting off at D with the Velocity it had before at O: And if G, the Seat of the retaining Power be at an infinite Diſtance, or in other Words (becauſe to ſay a determinate Point is at an infinite Diſtance, is a contradiction in Terms) if the retaining Power act in ſuch manner as it would do if the Seat of it was at an infinite Diſtance; that is, if it act upon the revolving Body in Lines parallel to DG; and with the ſame Degree of Force, whether the Body be at D, at P, or at Q, &c. it will deſcribe the Parabola DPQ with whatever Velocity it ſets off with from D. For to deſcribe a Circle round the Point G at an infinite Diſtance; it muſt ſet off with an infinite Velocity, becauſe, as obſerved above, it muſt ſet off with the Velocity it would acquire by falling half way to that Point; now a Body will deſcribe an Ellipſe round a given Point with any Degree of Velocity leſs then it will deſcribe a Circle with round the ſame, as it was obſerved that the Ellipſe DLN was deſcribed with leſs Velocity than the Circle DKM, but a Parabola is no other than an Ellipſe, one of whoſe *Foci* is at an infinite

Diſtance, the Body will therefore deſcribe a Parabola when the Seat of the retaining Power is at an infinite Diſtance, with any Degree of Velocity leſs than an infinite one.

This is the Reaſon, that Projectiles upon the Surface of the Earth are ſaid to deſcribe Parabola's, becauſe on Account of the great Diſtance of the Center of the Earth, compared with the Height Bodies can be thrown to, Gravity does as to Senſe, act upon them uniformly and in parallel Lines.

Again, when the Point G the Seat of the retaining Power goes off at an infinite Diſtance one way; it immediately, or indeed ſtrictly ſpeaking, at the ſame Inſtant, becomes the Point H at an infinite Diſtance the other way; ſo that thoſe two conſidered as at an infinite Diſtance, are as it were one and the ſame Point; and conſequently it is the ſame Thing whether the retaining Power be at G an infinite Diſtance to the *Right*, or at H an infinite Diſtance to the *Left*, and therefore the Figure will ſtill be a Parabola, and may alſo be deſcribed with any Degree of Velocity: only if it be at H, an infinite Diſtance to the left, the Power muſt be negative in reſpect of what it was before; that is, it muſt be of the *repulſive* Kind, as it is obvious it ought to be, to cauſe the revolving Body to deſcribe a Figure whoſe Convexity is turned towards the Seat of the retaining Power.

If the Seat of the retaining Power H comes nearer, ſo as to be at a finite Diſtance from the other Focus S, the Power muſt ſtill be negative, or repulſive; and the Figure deſcribed will be an Hyperbola, whoſe Foci are H and S. And the Hyperbola in this Caſe may be deſcribed whatever be the Velocity the Body ſets off with; for the central Force being of the repulſive kind, the Figure deſcribed will neceſſarily be convex towards the Seat thereof, that is, towards one of its Foci; but none of the Sections of the Cone have one of their Foci on the convex Side of the Figure, and at a finite Diſtance from its Vertex, except the Hyperbola *(h)*.

Hence

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*(h)* In order to evince the Truth of what is here affirmed, the following Lemma will be of Uſe.

### LEMMA X.

Let the Curve Line DPQ in Fig. 35. repreſent a conic Section, one of whoſe *Foci* is S, and let the Circle DKR be of the ſame Curvature therewith at its Vertex D, then if the Curve be a Parabola, DR the Diameter of the Circle will be equal to, if it be an Ellipſe, it will be leſs, if an Hyperbola it will be greater than four times the Diſtance DS.

*Demonſtrat. of the Lemma.* (By Milnes Conic. Sect. Part III. Prop. 2. Coroll. 5.) DR the Diameter of the Circle is equal to the Parameter of the Axis of the Curve, whether it be a Parabola, Ellipſe or Hyperbola, which let us call, L.

| | | |
|---|---|---|
| Therefore | 1 | $L = DR$ |
| 1. But in the Parabola, by the Definition of the Parameter, (De L'Hoſpit. Con. Sect. Livr. I. Def. 5.) | 2 | $L = 4DS$ |
| Comparing 1ſt and 2d Step | 3 | $DR = 4DS$. Q. E. D. |

2. If

| | | |
|---|---|---|
| 2. If the Curve be an Ellipſe, let F be its other Focus, then will DF + DS be equal to its longer Axis, De L'Hoſp. Con. Sect. Liv. 2. Cor. 2. of the Definitions; call its ſhorter Axis, X. Then from the Definition of the Parameter (De L' Hoſp. Conic. Sect. Liv. II. Def. 8.) we ſhall have this Proportion | 4 | $L : X :: X : DS + DF$ |
| Turning the laſt Step into an Equation | 5 | $L = \frac{X^q}{DS + DF}$ |
| (By De L' Hoſpit. Conic. Sect. Liv. 2. Corol. 4. of the Definitions) the Square of half X is equal to DS × DF, that is | 6 | $\frac{X^q}{4} = DS \times DF$ |
| Conſequently | 7 | $X^q = 4 DS \times DF$ |
| Comparing the fifth and ſeventh Step | 8 | $L = \frac{4 DS \times DF}{DS + DF}$ |
| Turning this Equation into a Proportion | 9 | $L : 4 DS :: DF : DF + DS$ |
| Comparing the 1ſt and the 9th Step | 10 | $DR : 4 DS :: DF : DF + DS$ |

But DF is leſs than DF + DS, therefore DR is leſs than 4 DS. Q. E. D.

3. If the Curve be an Hyperbola, let H be its other Focus, then will DH — DS be equal to its longer Axis, by De L'Hoſpit. Con. Sect. Liv. 3. Corol. 2. of the Definitions: And call as before its ſhorter Axis X; then from the Definition of the Parameter (De L'Hoſpit. Conic. Sect. Liv. 3. Defin. 8.) we ſhall

| | | |
|---|---|---|
| have this Proportion | 11 | $L : X :: X : DH - DS$ |
| Turning the laſt Step into an Equation | 12 | $L = \frac{X^q}{DH - DS}$ |
| (By De L' Hoſpit. Conic. Sect. Liv. 3. Coroll. 4. of the Definitions) the Square of half X is equal to DS x DH, that is | 13 | $\frac{X^q}{4} = DS \times DH$ |
| Therefore | 14 | $X^q = 4 DS \times DH$ |

PART IV. PLATE VII. P.234.

Fig. 25.

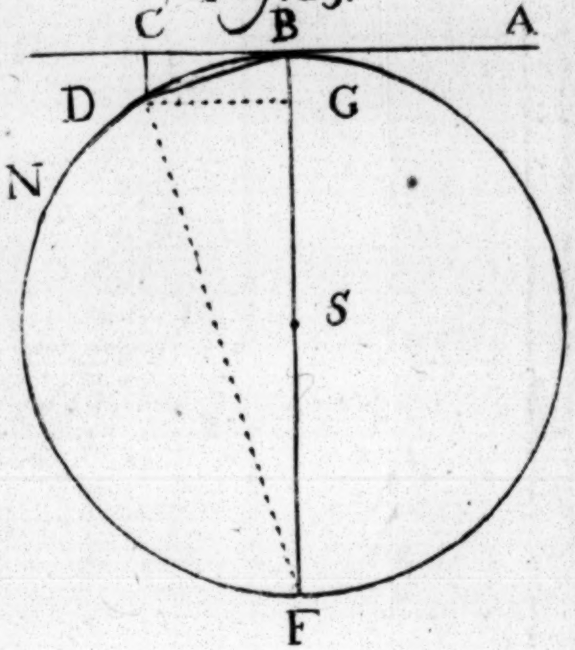


Fig. 26

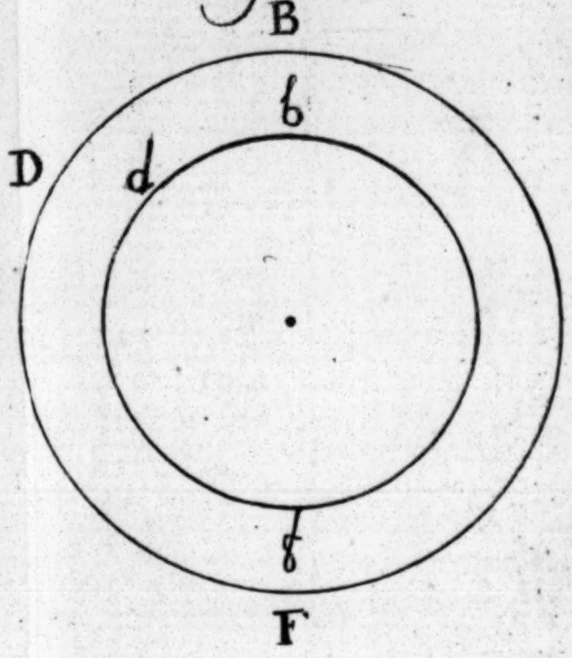


Fig 27.

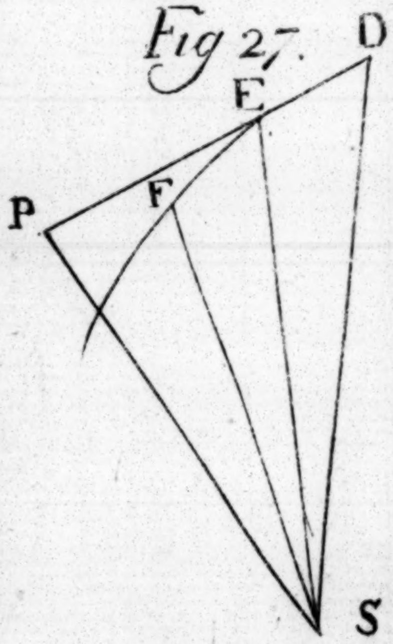
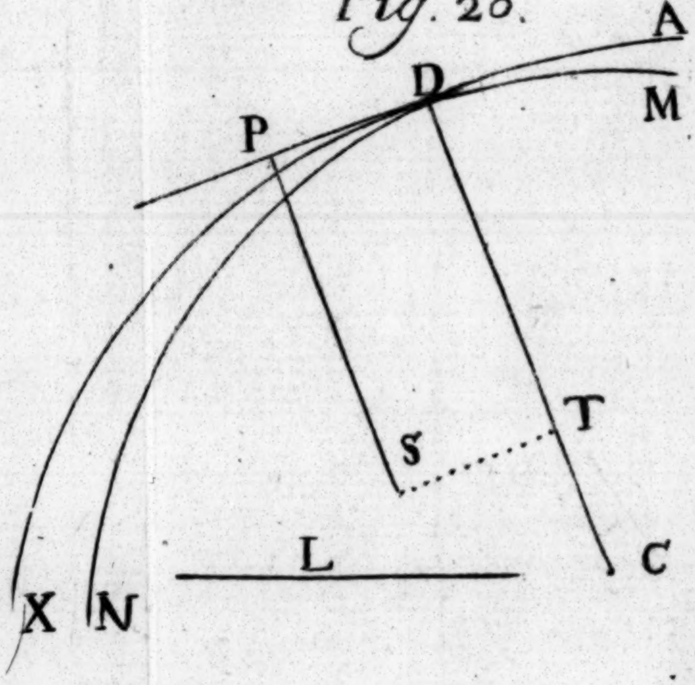


Fig. 28.



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|---|---|---|
| Comparing the 11th and 13th Steps | 14 | $L=\frac{4\,DS\times DH}{DH-DS}$ |
| Turning this into a Proportion | 15 | $L : 4DS :: DH : DH-DS$ |
| Comparing the 1ſt and 14th Steps | 16 | $DR : 4DS :: DH : DH-DS$ |

But DH is greater than DH—DS, therefore DR is greater alſo then 4 DS. Q. E. D. This being premiſed, we may proceed to confirm what was laid down in the Text above.

1. Let the Curve DPQ be a Parabola, and the Circle DKR being of the ſame Curvature with the Parabola at the Point D, as ſuppoſed above, the Velocity a Body ought to ſet off with from D, to deſcribe the Circle, is the ſame with that with which it ought to ſet off, to deſcribe the Parabola: but the Velocity it ought to ſet off with to deſcribe the Circle, is by Lemma 2. ſuch as it would acquire by falling to S, becauſe DS being (by Lemma 10.) a Quarter of DR, is equal to half the Radius of the Circle; call this Space 2: but the Velocity it ought to ſet off with to deſcribe a Circle, whoſe Center is S, is ſuch as it would acquire by falling through half DS, which Space (becauſe the whole Line DS is called 2) muſt be called one. Now the Velocities Bodies acquire by falling through certain Spaces are to each other as the Square Roots of thoſe Spaces (by Part I. Chap. 5. § 6.) the abovementioned Velocities therefore are to each other as $\sqrt{2}$ to 1. That is, the Velocity a Body ought to ſet off with from D (the Vertex of the Figure, and in a Direction perpendicular to the Axis DS, which I would always have ſuppoſed) in order to revolve in a Parabola whoſe Focus is S, is to that it ought to have at the ſame Place (and in the ſame Direction) to deſcribe a Circle whoſe Center is S, as $\sqrt{2}$ to 1.

2. Had the Figure DPQ been an Ellipſe, a Quarter of DR the Diameter of a Circle of the ſame Curvature therewith, had been (by Lemma 10.) leſs than DS; and ſo the Velocity the revolving Body muſt have had to deſcribe that Circle, or (which is the ſame Thing, becauſe of their equal Curvatures) the Ellipſe, would have been acquirable by falling through a leſs Space than DS, and therefore would have been a leſs Velocity. That is, it would have exceeded the Velocity requiſite for a Circle, whoſe Center is S the Seat of the retaining Power, in a leſs Proportion than that of $\sqrt{2}$ to 1.

3. If the Figure DPQ be an Hyperbola, a Quarter of DR is bigger than DS (by Lemma 10.) and ſo the Space a Body muſt fall through to get a competent Velocity for that Curve, will be greater than what it muſt fall through to obtain a competent Velocity for the Parabola: The Velocity therefore requiſite for

Hence we may obſerve, that, ſuppoſing the Motion of a revolving Body be ſo adjuſted to the Power by which it is retained, that it ſhall deſcribe a *circular* Orb, and its Velocity be afterwards altered by ſome Accident, and thereby the Adjuſtment thereof to the central Force deſtroyed, that Body ſhall not immediately fall to the Center: the Form of its Orbit only will be altered, and from being circular, will become elliptical, parabolical, or hyperbolical, according to the Alteration made in its Velocity. Let its Velocity be diminiſhed in any Degree whatever, or let it be increaſed, ſo it be in a leſs Proportion than as $\sqrt{2}$ to 1, its Orbit becomes an Ellipſe; if it be increaſed exactly in that Proportion, a Parabola; if in a greater, an Hyperbola. Hence we have a probable Reaſon why the Orbits of the Planets are now elliptical; for ſuppoſing them to have been circular at firſt, as it is not unlikely they were, yet upon the firſt Diſturbance in their Motions, whether from their mutual Action upon each other, the Reſiſtance of the Medium they revolve in, the Acceſs of a Comet, or any other Cauſe whatever; and whether their Motion be increaſed or decreaſed thereby, provided it be not increaſed in a De-

the Hyperbola exceeds the Velocity required for a Circle whoſe Center is S, in a greater Proportion than that of $\sqrt{2}$ to 1. From which all that is aſſerted above in the Text to which this Note refers, except what is there otherwiſe accounted for, may be collected.

gree beyond the Proportion above laid down, their Orbits would immediately become elliptical. Should the Motion of any of them be increaſed in or above the ſaid Proportion, its Orbit would accordingly be changed into a Parabola, or an Hyperbola, and the Planet would go off, never to return.

PROP. VIII. If the Force of the central Power decreaſes as the Square of the Diſtance increaſes, and ſeveral Bodies revolving about the ſame, deſcribe Orbits that are *elliptical*, the Squares of the periodical Times of thoſe Bodies will be to each other as the Cubes of their middle Diſtances from the Seat of that Power (*k*).

---

(*k*) To demonſtrate this Propoſition, let the following Lemma be premiſed.

LEMMA XI.

The Area a revolving Body deſcribes, is as its Velocity multiplied by a Perpendicular let fall from the Seat of the retaining Power to a Tangent at that Point of the Curve where the Body is.

*Dem. of the Lemma.* Let AB in Fig. 36. be the Curve deſcribed, DC a Portion of the Curve deſcribed in the leaſt Time poſſible, then will DC repreſent the Velocity of the Body; produce DC to P, then will DP be a Tangent to the Curve: From S the Seat of the retaining Power let fall the Perpendicular SP, draw DS and CS; then will DCS be the Area the Body deſcribes, but DC by reaſon of its ſhortneſs may be conſidered as a ſtraight Line, the Area DCS is therefore a Triangle, which is had by multiplying the Baſe DC into half the Perpendicular SP, therefore it is proportionable to DC multiplied by the whole Line SP, that is, to the Velocity of the revolving Body multiplied by a Perpendicular let fall from the Seat of the retaining Power to a Tangent at that Point of the Curve where the Body is. Q.E.D.

*Dem. of the Propoſition.* Let ADB and *adb* in Fig. 37 and 38. repreſent two Ellipſes, whoſe principal Parameters are L and L, and let S be one of the Foci of the one, and *s* one of thoſe of the other, and let the Ellipſes be ſuppoſed to lie in ſuch manner upon one another, that the Points S and *s* may be one and the ſame Point, which let us ſuppoſe to be the Seat of the retaining Power; and let the revolving Bodies be at D and *d*, through which draw the Tangents PD and *pd*, the Perpendiculars PS and DC, and *ps* and *dc*, and the Lines DS and *ds*; and let DC and *dc* be Radii of Curvature to the Points D and *d*; and let the Force of the retaining Power at the Diſtance SD be called F, and at the Diſtance *sd* be called *f*, and let the Velocities of the revolving Bodies when at the Points D and *d* be called V and *v*. Then becauſe by the Propoſition the Force of the central or retaining Power is ſuppoſed to decreaſe as the Squares of the Diſtances SD and *sd* increaſe, that is, to be reciprocally as the Squares of thoſe Diſtances, we ſhall have for the firſt Step of the following Proceſs, this Analogy, *viz.* | 1 | $F : f :: \frac{1}{SDq} : \frac{1}{sdq}$ Parallel to the Tangents draw the Lines SH and *sh*, and let the Force of the retaining Power which acts upon the Body at D in the Direction DS be reſolved into two others, *viz.* DP and DH, the former along the Tangent DP, the latter along the Radius of Curvature DC, then it is obvious that it is by this latter Force only that the revolving Body is retained in the Circle whoſe Radius is DC, that is, in the Ellipſe at the Point D, the other Force along the Tangent only accelerates the Body when moving towards A, and retards it if moving towards B. To compute the Quantity of Force in the Direction DC, ſay as DS is to DH ſo is the oblique Force F, or inſtead thereof $\frac{1}{SDq}$ (ſee Step the 1ſt) to a fourth Number, which fourth Number by the Golden Rule will be $\frac{DH}{SD^3}$, or (becauſe SP is equal to DH) $\frac{SP}{SD^3}$; and in like manner we ſhall have $\frac{sp}{sd^3}$ for the Force retaining the other Body in the Circle whoſe Center is *c*; but by Lemma 3. the Force neceſſary to retain a Body in a Circle whoſe Radius is CD is expreſſible by the Square of its Velocity divided by the Radius of the Circle it deſcribes, that is, in the preſent Caſe by $\frac{Vq}{CD}$. And in like manner the Force retaining the other Body in its Circle, will be $\frac{vq}{cd}$, conſequently | 2 | $\frac{SP}{SD^3} : \frac{sp}{sd^3} :: \frac{Vq}{CD} : \frac{vq}{cd}$

Multiplying

| | | |
|---|---|---|
| Multiplying the laſt Step by CD : *cd* | 3 | $\frac{SP}{SD^3}\times CD : \frac{sp}{sd^3}\times cd :: V^q : v^q$ |
| Since CD is ſuppoſed to be the Radius of a Circle of equal Curvature with the Figure, we ſhall have by Lemma 9. theſe Equations, *viz.* | 4 | $CD = \frac{L\times SD^3}{2\,SP^3}$, and $cd = \frac{l\times sd^3}{2\,sp^3}$ |
| Comparing the 3d and 4th Steps | 5 | $\frac{SP}{SD^3}\times\frac{L\times SD^3}{2\,SP^3} : \frac{sp}{sd^3}\times\frac{l\times sd^3}{2\,sp^3} :: V^q : v^q$ |
| That is, by reducing the laſt Step to lower Terms | 6 | $\frac{L}{SP^q} : \frac{l}{sp^q} :: V^q : v_q$ |
| Extracting the Square Root of each Term | 7 | $\frac{\sqrt{L}}{SP} : \frac{\sqrt{l}}{sp} :: V : v$ |
| Multiplying by SP : *sp* | 8 | $\sqrt{L} : \sqrt{l} :: V\times SP : v\times sp$ |
| But by Lemma 2. the Area one Body deſcribes is to that which the other deſcribes in the ſame time, as V × SP to *v* × *sp*, which Areas call A and *a*, and we ſhall have this Proportion, *viz.* | 9 | $A : a :: V\times SP : v\times sp$ |
| Comparing the 8th and 9th Steps | 10 | $A : a :: \sqrt{L} : \sqrt{l}$ |

Now the Area of the whole Ellipſe ADB is to that of the other *adb*, as AB the longer Axis multiplied by QN the ſhorter, to *ab* × *qn*, thoſe Quantities therefore may be put for the Areas themſelves: If then we call the Time in which the Areas A and *a* are deſcribed in, one, (as one Hour, or one Minute ſuppoſe) and the Times the whole Ellipſes are deſcribed in, T and *t*; we ſhall have (ſince the Area a revolving Body deſcribes is proportional to the Time it is deſcribed in) the following Analogies,

| | | |
|---|---|---|
| *Viz.* | 11 | $1 : T :: A : AB\times QN$ |
| And | 12 | $1 : t :: a : ab\times qn$ |

Comparing

| | | |
|---|---|---|
| Comparing the 3 laſt Steps together we have the two following ones, *viz.* | 13 | $1 : T :: \sqrt{L} : AB \times QN$ |
| | 14 | $1 : t :: \sqrt{l} : ab \times qn$ |
| By De L'Hoſpital Conic. Sect. Liv. 2. Def. 8. | 15 | $AB : QN :: QN : L$ |
| Turning the laſt Proportion into an Equation. | 16 | $AB \times L = QN^{q}$ |
| Extracting the Square Roots of each Side of the Equation | 17 | $\sqrt{AB} \times \sqrt{L} = QN$ |
| And in like manner | 18 | $\sqrt{ab} \times \sqrt{l} = qn$ |
| Comparing the 13th and 17th Steps together | 19 | $1 : T :: \sqrt{L} : AB \times \sqrt{AB} \times \sqrt{L}$ |
| And comparing the 14th and 18th together | 20 | $1 : t :: \sqrt{l} : ab \times \sqrt{ab} \times \sqrt{l}$ |
| Dividing the latter Part of the 19th Proportion by $\sqrt{L}$ | 21 | $1 : T :: 1 : AB \times \sqrt{AB}$ |
| Dividing the latter Part of the 20th Proportion by $\sqrt{l}$ | 22 | $1 : t :: 1 : ab \times \sqrt{ab}$ |
| Comparing the 21ſt and 22d | 23 | $T : t :: AB \times \sqrt{AB} : ab \times \sqrt{ab}$ |
| Squaring each Term | 24 | $TT : tt :: AB^{q} \times AB : ab^{q} \times ab$ |
| That is | 25 | $TT : tt :: AB^{3} : ab^{3}$ |

But half the longer Axis AB is equal to the middle Diſtance SN, (De L'Hoſpital Conic. Sect. Liv. 2. Corol. 3. of the Definitions) therefore ſince Halves are proportionable to their Wholes

26 $TT : tt :: SN^{3} : sn^{3}$. Q. E. D.

Prop. IX. If the retaining Power decreaſes ſomething faſter as you go from the Seat thereof (or which is the ſame thing, increaſes ſomething faſter, as you come towards it) than in the Proportion mention'd in the laſt Propoſition, and the Figure the revolving Body deſcribes be not a Circle, the Axis of that Figure will turn the ſame Way the Body revolves; but if the ſaid Power decreaſes (or increaſes) ſomewhat ſlower than in that Proportion, the Axis of the Figure will turn the contrary Way. Thus, if a revolving Body as D, Fig. 39. paſſing from A towards B deſcribes the Figure ADB, whoſe Axis AB at preſent points towards M and N, and the Power whereby it is retained decreaſes faſter than the Square of the Diſtance increaſes, after a Number of Revolutions the Axis of the Figure will point towards O and P, and after that towards Q and R, &c. revolving round the ſame Way with the Body, and if the retaining Power decreaſes ſlower than in that Proportion, the Axis will turn the other Way. (*i*)

---

(*i*) *Dem.* Let S in Fig. 40. repreſent the Seat of the retaining Power, and let a Body as D, deſcribe the Orb ADB paſſing from A towards B; and let another as N deſcribe a revolving Orb as MNO equal and ſimilar to the former; and let both the Orbits be deſcribed in the ſame Time; and let the Velocity wherewith the Orb MNO revolves be ſuch, that the Angle ASN ſhall always be proportionable to the Angle ASD. To explain this a little farther, ſuppoſe that, when the Bodies N and D ſet out from M and A, thoſe two Points were then together at A; or that the Orbits did then coincide; but that N advances forwards in ſuch Manner, as that if the Angle ASN is at any time double, treble, or in any other Proportion of the Angle ASD, it ſhall always be ſo: That is, that the Line SN ſhall always move faſter or ſlower than the Line SD in ſome certain Proportion; and that the Point M ſhall follow or go from the Body N, ſo as that the Arch MN ſhall always be equal to the Arch AD. And let it farther be ſuppoſed that the Line SN is always equal to SD.

The firſt Inquiry which ariſes here is, whether this be a poſſible Caſe; that is, whether the Body N moving with the above-mentioned Reſtrictions, the Line SN can deſcribe Areas proportionable to the Times, becauſe as was ſhewn, Part. I. Chap. 8. that is always requiſite where Bodies revolve round a central Force, and are retained in their Orbits thereby. Let it then be conſidered in the firſt Place, that the Body D revolving in the quieſcent Orb ADB may do ſo, its Motion being clogged with none of thoſe Suppoſitions; ſecondly, that the Line NS is always equal to DS, and the Velocity wherewith its Extremity N moves forwards, is proportionable to that which D the Extremity of the other moves with; and conſequently the Areas which one deſcribes, (for the Areas depend ſolely on the Length and Velocity of the Lines that deſcribe them) will be proportionable to thoſe which the other deſcribes: Since then thoſe of the former may be proportionable to the Times they are deſcribed in, it is poſſible thoſe of the latter may be ſo too; the Suppoſition therefore above laid down is not abſurd.

The next Inquiry is, by what Law the Action of the central Force at S muſt decreaſe as we recede from it, that a Body as N may deſcribe the revolving Orb MNO in the manner abovementioned Or, which will anſwer our Purpoſe as well, how the Force neceſſary to retain the Body N deſcribing the revolving Orb MNO, muſt differ from that which is requiſite to retain the other Body D deſcribing the quieſcent Orb ADB equal and ſimilar thereto.

In order to this, let DR, in Fig. 41. repreſent a Portion of the quieſcent Orb deſcribed in the leaſt Time poſſible, this then may be conſidered as a ſtrait Line; and let the Motion of the Body along this Line be reſolved into two, the one towards S along the Line DT, and another along the Line TR at right Angles therewith; then will DT repreſent the Velocity wherewith the Body deſcends towards S, and conſequently the Action of the central Force whereby it is retained in that Arch; and TR will repreſent that by which it advances forwards in the mean time. Let it now be ſuppoſed that the Orb ADB advances forwards, while the Arch DR is deſcribing; but not by Virtue of any additional Force in the Center, but by ſome extrinſic one, no matter what: That

is,

PART IV. PLATE VIII. *Pag.* 242.

*Fig.* 29.

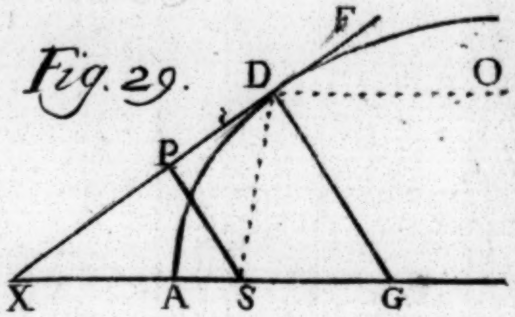


*Fig.* 30.

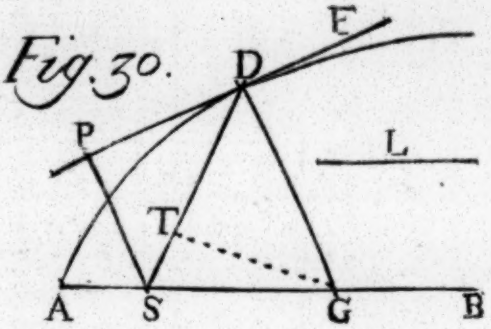


*Fig.* 31.

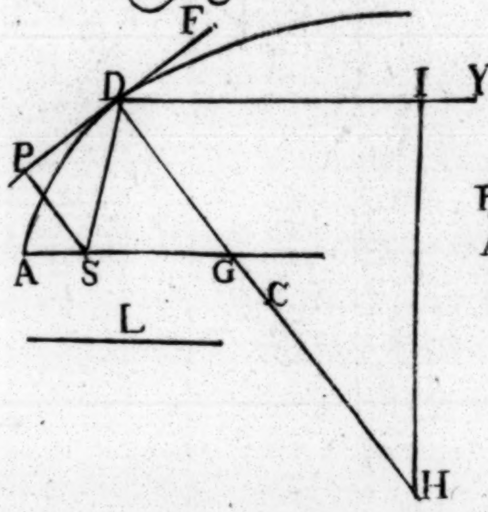


*Fig.* 32.

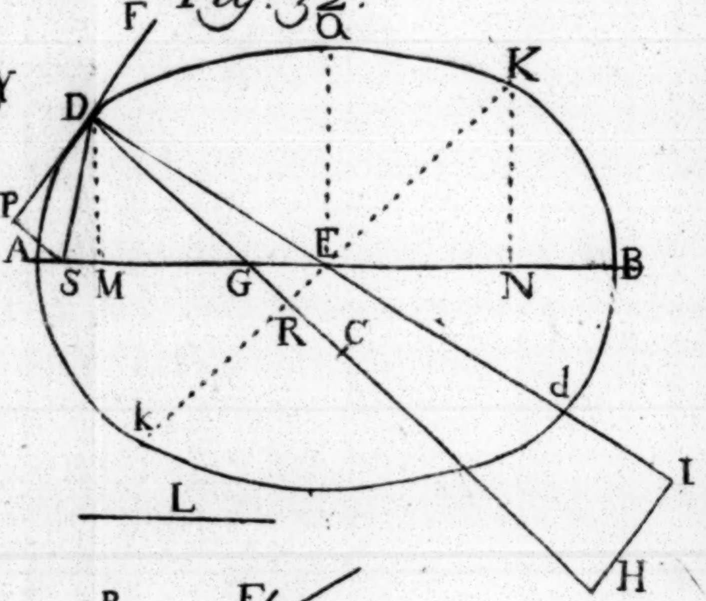
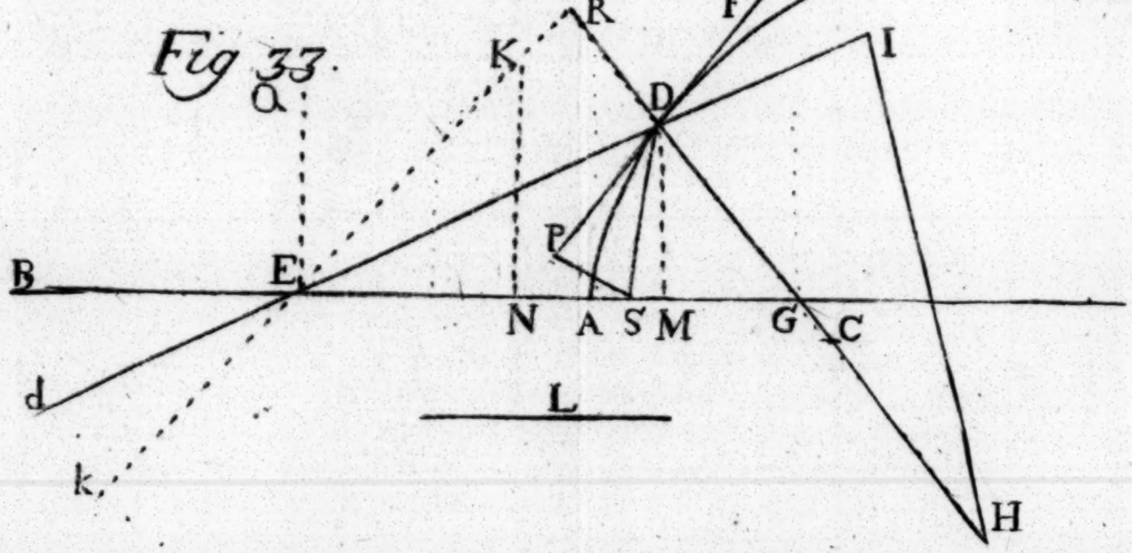


*Fig* 33.



is, ſuppoſe that the Line SD turns round the Point S faſter than it would otherwiſe have done, carrying the Body with it ſo much the faſter; but that the Body deſcends towards S along that Line juſt as it did before: That is, in other Words, that the Line DT which repreſents the Deſcent of the Body, is of the ſame Length as before; but that the other Line TR which repreſents the Progreſs of the Body forwards, is longer than it was in the other Caſe. Let it then become TQ; in which Caſe, the Body moving over the Line DT and the Line TQ in the ſame Time, will in reality get to the Point Q: But obſerve, that the Point Q is farther from S than the Point R is, becauſe the Angle at T is a right one; whereas the Law which we laid down above for the Motion of a Body deſcribing a revolving Orb was ſuch, that at the End of the Time in which DR would be deſcribed in the quieſcent Orb, the Body in the revolving Orb, ſhould be at the ſame Diſtance from S as if its Orb had not revolved; if then round the Point S, we deſcribe a Circle as RFG, and thro' S draw the Line QG, N muſt be the Point the Body muſt come to. Since then the Body in deſcribing the revolving Orb without any Addition to the central Force above what would cauſe it to deſcribe the quieſcent one, would come to Q; but with ſuch an Addition as is neceſſary to make it revolve in the manner above laid down, it comes to N, it is obvious that the Line QN being the Diſtance between thoſe two Points will aptly repreſent that Addition. It remains then to get the Meaſure of that Line, which may be done in the following manner.

Produce QT to F, then (by 35. El. 3.) will the Rectangle QN × QG be equal to QR × QF. Therefore

| | | |
|---|---|---|
| | 1 | $QN \times QG = QR \times QF$ |
| Dividing by QG | 2 | $QN = \frac{QR \times QF}{QG}$ |

Let it be obſerved then in the firſt Place, that as the Motion of revolving Bodies is ſuch that they deſcribe equal Areas in equal Times, the longer the Line DS is (that is the nearer D is to A, ſuppoſe) the ſhorter the Line TR muſt be, and for the ſame Reaſon alſo the Line TQ: That is, both TR and TQ are reciprocally as SD, which let us put thus

| | | |
|---|---|---|
| | 3 | $TR : \frac{1}{SD}$ |
| And | 4 | $TQ : \frac{1}{SD}$ |
| Obſerve ſecondly, that ſince RF is a Chord of a Circle, and ST perpendicular to it | 5 | $RT = TF$ |

From the Figure | 6 | $QR = TQ - TR$

Becauſe RT and TF are equal | 7 | $QF = QT + TR$

Since then by the 3d and 4th Steps, TR and TQ are both reciprocally as SD, and by the 6th and 7th Steps, QF is equal to the Sum of thoſe Lines, and QR is equal to their Difference, it is obvious that QF and QR are alſo each reciprocally as SD,

which put thus | 8 | $QF : \frac{1}{SD}$

And | 9 | $QR : \frac{1}{SD}$

Multiplying the 8th and 9th Steps together | 10 | $QR \times QF : \frac{1}{SD^q}$

Obſerve alſo that when the Points D, T, R, Q and N coincide, which muſt ever be ſuppoſed, becauſe we are all this Time conſidering only what is done at one and the ſame Point of the Curve ADB, QN is nothing with reſpect to NG, therefore QG and NG are the ſame Line; therefore QG may be ſaid to be as NG, or as half of it, *viz.* RS, or which will be the ſame Thing, DS; therefore inſtead of QG in the 2d Step, we may put DS,

and then it will ſtand thus | 11 | $QN = \frac{QR \times QF}{DS}$

The 11th Step without altering the Value of it, may be ſet thus | 12 | $QN = QR \times QF \times \frac{1}{DS}$

Comparing the 10th and 12th Steps together we have | 13 | $QN = \frac{1}{DS^q} \times \frac{1}{DS}$

That is | 14 | $QN = \frac{1}{DS^3}$.

That is, the Line QN, or the additional central Force requiſite to cauſe a Body to move in a revolving Orb muſt be reciprocally as the Cube of the Diſtance of the revolving Body from the Seat of the retaining Power. Whereas then, when the centripetal Force decreaſes as the Square of the Diſtance increaſes, the revolving Body deſcribes a Parabola, an Ellipſe, or an Hyperbola; if to that centripetal Force be ſuperadded another, that ſhall decreaſe as the Cube of the ſaid Diſtance increaſes; thoſe two Forces acting conjointly upon a revolving Body ſhall cauſe it to deſcribe the ſame Figure as before, but the Axis thereof ſhall revolve the ſame Way that the Body does.

But obſerve here, that if a Force decreaſes faſter in any Degree whatever than the Square of the Diſtance increaſes, but ſlower than the Cube, that Force is the Sum of two Forces, one of which decreaſes as the Square, the other as the Cube of the Diſtance

Thus it is the heavenly Bodies, *viz.* the Planets both primary and ſecondary, and alſo the Comets, perform their reſpective Revolutions. The Figures in which the primary Planets and the Comets revolve, are Ellipſes, one of whoſe Foci are at the Sun. The Areas they deſcribe by Lines drawn to the Center of the Sun, are in each proportionable to the Times in which they are deſcribed. The Squares of their periodical Times are to each other as the Cubes of their middle Diſtances from the Sun. The ſecondary Planets deſcribe alſo Circles or

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Diſtance increaſes. Thus for Inſtance, let the Diſtances be as 2 to 1, the Squares of them will be 4 to 1, their Cubes 8 to 1; and let the Forces be as 6 to 1; I ſay, this Proportion will ariſe from adding 4 to 1 to 8 to 1; for 4 to 1 added to 8 to 1, is 12 to 2, that is 6 to 1. Again, let the Forces be as 5 to 1; I ſay, this alſo will ariſe from adding 4 to 1 to 8 to 1; for inſtead of 4 to 1 let us take 12 to 3 which is the ſame Thing, this ſuperadded to 8 to 1 makes 20 to 4, that is, 5 to 1. Again, let the Forces be as 7 to 1; this alſo is the Sum of 4 to 1 added to 8 to 1; for inſtead of 8 to 1 we may take 24 to 3, which added to 4 to 1 gives 28 to 4, or 7 to 1. And the ſame for Fractions: ſo that let a Proportion be what it will between 4 to 1 and 8 to 1, it may be conſidered as ariſing from 8 to 1 added to 4 to 1. When the retaining Power therefore by which a revolving Body is kept in its Orbit, decreaſes faſter than the Square of the Diſtance increaſes, but not ſo faſt as the Cube thereof does; it is a Power decreaſing as the Square, having another Power decreaſing as the Cube of the Diſtance ſuperadded to it; and therefore as the Propoſition in the Text aſſerts, if the retaining Power decreaſes ſomething faſter, *&c.* Q. E. D. As to the Figures turning the contrary Way, when the centripetal Forces decreaſe ſomewhat ſlower than as the Square of the Diſtance increaſes; that is but the Converſe of this, and therefore needs no particular Demonſtration.

Ellipſes, one of whoſe Foci are in the Center of their Primary ones. The Area that each deſcribes by Lines drawn to the Center of its Primary, is proportionable to the Time it is deſcribed in. The Squares of the periodical Times of the Secondaries belonging to the ſame Primary, are to each other as the Cubes of their Diſtances from their Primary. And above all, that very Force by which Stones or heavy Bodies fall to the Surface of the Earth with us, is at the Diſtance of the Moon, juſt ſufficient to retain her in her Orbit, ſuppoſing it to decreaſe as we go from the Center of the Earth in that Proportion, in which the Cauſe, that retains her and the reſt of the Planets in their Orbs, whatever it be, muſt do to make them elliptical *(l)*. And farther, which alſo is a neceſſary

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*(l)* To find out whether this be ſo or not, let us calculate what Velocity the Moon would acquire, were ſhe to fall half way to the Center of the Earth by Virtue of that Gravity, whereby heavy Bodies tend to the Earth; and compare it with the Velocity ſhe moves with in her Orbit; becauſe by Lemma 2 of this Chapter, if ſhe be retained in her Orb by that Gravity, thoſe Velocities ought to be the ſame. The mean Diſtance of the Moon from the Earth in round Numbers is 60 Semidiameters of the Earth, therefore the Force of Gravity at the Diſtance of the Moon is the Square of 60 times leſs than it is at the Surface of the Earth; therefore the Fall of a Body at that Diſtance in a Second of Time, will be ſo many times leſs than it is here; but the Fall of a Body here in a Second, is about 15 *Paris* Feet, conſequently at the Moon it is but ,004166 Feet. The Space therefore it would move over in a Second, with the Velocity acquired by that Fall, is, by Part I. Chap. 5. § 7. twice that Quantity, *viz.* ,008332 Feet, which Number therefore may be put to expreſs its Velocity. Now becauſe the Velocities Bodies acquire

neceſſary Conſequence of what we are about to lay down, the Motion of the Planet *Saturn* is obſerved to be diſturbed by *Jupiter*; and the Secondaries of *Jupiter* to be diſturbed in their Motions upon the nearer Approach of *Saturn*; and the Courſe of the Moon is inceſſantly altered by the Action of the Sun, in ſuch manner as to cauſe all thoſe Irregularities mentioned in Chapter the 8th of this Part. All which, together with the Preceſſion of the Equinoctial Points, the Nutation of the Poles of the Earth, and the Phænomena of the Tides, which naturally flow from it, make it extremely probable that there is a Virtue diffuſed about the Sun and Planets, not unlike that of Attraction, which decreaſes as the Squares of the Diſtances from the Centers of thoſe Bodies increaſe, and

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acquire by falling, are as the ſquare Roots of the Spaces they fall through, Part I. Chap. 5. § 6. ſay, as the Square Root of ,004166, is to the Square Root of 590866170, which is half the Semidiameter of the Moon's Orbit, (ſuppoſing the Semidiameter of the Earth to be 19695539 Feet, which is *Caſſini*'s Meaſure, and the Diſtance of the Moon from the Earth to be 60 Semidiameters as above) ſo is ,008332 to 3135 Feet, which is therefore the Space the Moon would move over in a Second with the Velocity ſhe would acquire by falling half way to the Center of the Earth. But this comes within an hundredth Part of her Velocity in her Orbit, as may eaſily be calculated from the Time ſhe revolves in, *viz.* 27 Days, 7 Hours and 43 Minutes, and the Semidiameter of her Orbit, which according to the Meaſures we have taken above, is 1181732340. That Force therefore by which Bodies fall to the Ground with us, is at the Diſtance of the Moon ſuch as is requiſite to direct her Courſe.

*N. B.* The Proportion of a *Paris* Foot to that of *England*, is as 367196 to 343800.

acts upon Bodies in Proportion to the Quantity of Matter they contain.

This being allowed, it will follow, that as the Sun attracts the Planets, and thereby retains them in their Orbs, they in like manner attract the Sun, though with Forces proportionable only to the Quantities of Matter they contain; ſo that ſtrictly ſpeaking, each primary Planet revolves not about the Center of the Sun, but about a Point which is the Center of Gravity between the Sun and that; and that the Sun moves alſo round that Point, and is always oppoſite to the Planet with reſpect thereto: And likewiſe that the Center of the ſolar Syſtem is not in any one Body, but in the common Center of Gravity of all the Bodies of which it conſiſts. But then the Sun is ſo immenſely large in reſpect of any one, or all of thoſe Bodies put together, that that Center is very near the Center of the Sun. In like manner, the Moon does not revolve about the Center of the Earth as a Point at reſt, but the Earth and Moon revolve each about the Center of Gravity common to them both, which Center of Gravity it is, and not the Center of the Earth, that deſcribes the Orbit the Earth is commonly ſaid to revolve in. And ſo of the other Planets which are attended by Secondaries.

The Irregularities in the Motion of the Planet *Saturn*, and thoſe of the Secondaries of *Jupiter*, owing to the Cauſe abovementioned, are ſo exceeding ſmall, that it ſhall ſuffice to have juſt mentioned them.

The more remarkable Effects of the diſturbing Force of the Sun, are the lunar Irregularities, the Preceſſion of the Equinoctial Points, the Nutation of the Poles of the Earth, and the ebbing and flowing of the Sea, which ſhall be particularly conſidered in the following Chapter.

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## CHAP. XIX.

## *Of the* Lunar Irregularities, *the* Preceſſion *of the* Equinoctial Points; *the* Nutation *of the* Poles *of the* Earth, *and the* ebbing *and* flowing *of the* Sea.

I. TO account for the lunar Irregularities, let S in Fig. 42. repreſent the Sun, T the Earth, and LMNO the Orbit of the Moon, and let the Moon be in one of its Quadratures at L, and let the Lines LS and TS be drawn. It is obvious, that the Tendency the Moon has towards the Sun is along the Line LS, and that which the Earth has, is along the Line TS: Let then the former of theſe be reſolved into two others, the one along LA parallel and equal to TS, the other from L to T along the Line LT. The former of theſe Tendencies being parallel and equal to that whereby the Earth tends along the Line TS, alters not the Situation of the two Bodies L and T with reſpect to each other; that is, it diſturbs not the Motion of the Body L; but the other along LT increaſes its Tendency towards T.

And this Increaſe will be to the Tendency the Moon has to A, which is the ſame the Earth has to S, as the Diſtance LT to LA, or TS. Or in other Words, the Gravity of the Moon towards the Earth in the Quadratures is augmented by the Action of the Sun; and that Augmentation is to the Tendency the Earth has to the Sun, as the Length of the Line LT, or the Diſtance of the Moon from the Earth, to TS the Diſtance of the Earth from the Sun.

So that the greater the Moon's Diſtance is from the Earth, the Diſtance of the Sun remaining the ſame, the greater will this increaſe of the Moon's Gravity towards the Earth be. But if the Diſtance of the Moon from the Earth remains the ſame, and the Diſtance of the Sun be augmented, this additional Increaſe will be the leſs in Proportion to the Cube of that Diſtance *(a)*.

Let

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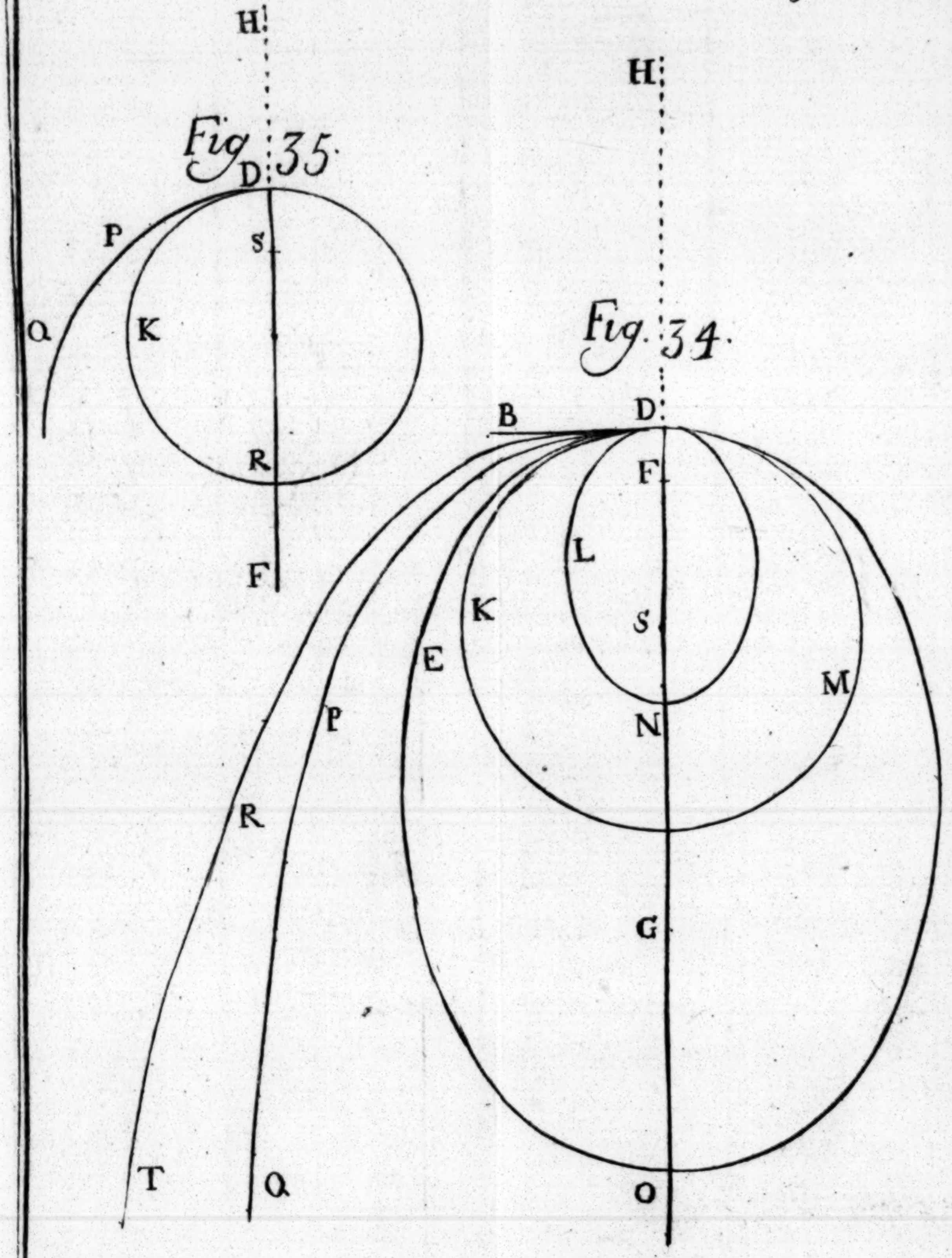
*(a)* For, if TS be increaſed, while LT remains the ſame, LT will be ſo much the leſs with reſpect to TS, that is the Increaſe will be diminiſhed in Proportion to the Sun's Diſtance: But

PART IV. PLATE IX. *Pag.* 250.



Let now the Moon be in one of its Syzygies at M, then will the Tendency ſhe has to the Sun over and above what the Earth has, which is farther off at T, be to that which the Earth has, as the Difference of the Squares of SM and ST is to the Square of SM: but the Difference between the Squares of SM and ST bears ſuch Proportion to the Square of SM as twice MT, that is MO, does to SM; becauſe the Difference between the Squares of two Numbers that are nearly equal (as SM and ST are on account of the great Diſtance of S) bears double the Proportion to the Square of the leſſer Number, that the Difference between the Numbers themſelves bears to the leſſer Number (*m*). The Tendency therefore the

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But when TS the Diſtance of the Sun is increaſed, the abſolute Force of the Sun, and therewith the abovementioned Increaſe, will be diminiſhed alſo in proportion to the Square of that Diſtance, conſequently taking in both the Accounts, it will upon the whole, be diminiſhed in Proportion to the Cube of that Diſtance.

(*m*) *Dem.* Let $a$ be the leſſer Number, and $a + b$ the larger, and let their Difference $b$ bear no ſenſible Proportion to the leſſer $a$.

Then the Square of the leſſer is ——— $aa$
The Square of the larger is ——— $aa + 2ab + bb$
The Difference between theſe is —— $2ab + bb$
And $bb$ being rejected as inconſiderable, the Difference is only } $2ab$
Now the Proportion of $2ab$ to $aa$ is (taking $a$ out of each Term) } $2b$ to $a$.

But $2b$ to $a$, is double the Proportion that once $b$ has to $a$, and therefore the Difference between the Squares of the two Numbers bears double the Proportion to the Square of the leſſer Number, that the Difference between the Numbers themſelves bears to the leſſer Number, Q. E. D.

Moon when at M, has to the Sun, over and above what the Earth has, is to that which the Earth has, as MO, or twice LT, to SM, or becauſe of the Sun's great Diſtance, as twice LT to TS. Her Tendency therefore to the Earth is now diminiſh'd in that Proportion: But as was ſhewn above, it was augmented in the Quadratures in the Proportion only of LT to TS. The Diminution here is therefore double of the Augmentation there.

And whereas that Augmentation, when the Diſtance of the Sun remains the ſame, was ſhewn to increaſe with the Diſtance of the Moon; but when the Diſtance of the Moon remains the ſame, to decreaſe with the Cube of the Sun's Diſtance; this Diminution being always double of that, will do the ſame.

When the Moon is in the other Syzygy at O, ſhe is attracted towards the Sun leſs than the Earth is by the Difference of the Squares of SO and ST; which as to the Effect, is the ſame Thing as though the Earth was not attracted at all towards S, and the Moon were attracted the contrary Way, ſo that her Tendency to the Earth is here alſo diminiſhed, as well as when ſhe was at M, and almoſt in the ſame Degree; for on Account of the Sun's great Diſtance, the Difference between the Spuares of SO and ST is nearly the ſame as between ST and SM.

Or becauſe this way of accounting for the Diminution of the Gravity of the Moon towards

wards the Earth in the Syzygies may not be ſufficiently clear, it may be conſidered otherwiſe, thus. The annual Courſe of the Moon round the Sun being performed in the ſame Time that the Earth's is, ſhe ought to be retained in that Courſe by the ſame Force that the Earth is, whereas when ſhe comes to M, the Action of the Sun upon her is greater than it is upon the Earth, by the Difference of the Squares of SM and ST; and when ſhe is at O, it is leſs than it is upon the Earth by the Difference between the Squares of ST and SO: So that in the former Caſe ſhe is drawn too much towards the Sun, and in the latter too little; and therefore in both Caſes her Tendency towards the Earth is diminiſhed; and almoſt in the ſame Degree; becauſe, as was obſerved above, the Difference of the abovementioned Squares is nearly the ſame in either Caſe.

Let the Moon be in a Point of her Orbit between the Quadrature and the Syzygy, as at L in Fig. 43. Then being nearer the Sun than the Earth is, ſhe will be attracted with a ſtronger Force: Let it be expreſſed by LS produced to D 'till LD is of ſuch Length, that TS being put to expreſs the Action of the Sun upon the Earth, LD may be long enough to expreſs the ſtronger Force of the Sun upon the Moon: And let LD be reſolved into two others, one of which let be LA equal and parallel to TS, then will the other be AD, or its equal and parallel LG. This LG is the only diſturbing Force upon the Moon at L, the other LA being parallel and equal to TS, affects the Moon juſt as the Sun does the Earth; and ſo alters not their Situations with reſpect to each other. Let then, to avoid a Confuſion of Lines, this Figure with the Line LG, be removed to the 44th. This Force LG may be reſolved into LI and LH, the one a Tangent to the Orbit of the Moon, and the other perpendicular thereto: The former accelerates the Motion of the Moon when going from the Quadrature at Q to the Syzygy at B; and will retard it when going from B to R. The other when H falls upon TL produced, as in this Figure, diminiſhes the Tendency of the Moon towards the Earth, and when it falls between L and T, it augments it.

When the Moon is at L between the Quadrature R and the oppoſite Syzygy O, in Fig. 45. the Tendency of the Moon towards the Sun is leſs than that of the Earth; it may then be expreſſed by LD a Line ſhorter than TS, and is reſolvable into LA a Line equal and parallel to TS, and DA, or its equal and parallel LG. Which LG is the only diſturbing Force, and may, as in Fig. 44. be reſolved into two others, one of which ſhall draw the Moon towards O, the other to or from T, as the Caſe may happen. So that in the firſt Place, the nearer the Moon is to its Syzygies, the greater will be its Velocity; and the nearer it is to the Quadratures, the ſlower it will move;

becauſe one of the Forces into which LG is reſolvable (as LI in Fig. 44.) accelerates its Motion from the Quadratures to the Syzygies; and retards it as much from thence to the Quadratures. *Which is the firſt Irregularity* (*n*).

2. When the Moon is in the Quadratures as at L or N, or in the Syzygies as at M or O, ſee Fig. 42. the diſturbing Force is directed to or from the Center of the Earth; and therefore when the Moon is paſſing thoſe Points it is no Impediment to her deſcribing Areas proportionable to the Times; but when ſhe is at L in Fig. 44, or 45, where it is expreſſed by LG, only one of the Lines into which it is reſolved, as LH, points to or from the Center of the Earth, the other, as LI, pointing another Way, prevents her deſcribing Areas proportionable to the Times. So that it is only in the Quadratures and Conjunctions, that the Areas are proportionable to the Times. *Which is the ſecond Irregularity.*

3. The Motion of the Moon being accelerated during her Progreſs from the Quadratures to the Syzygies, and retarded from thence to the Quadratures, her Motion in the Syzygies is too quick, in the Quadratures too ſlow: add to this, her Tendency to the Earth is in the former Situation too ſmall, in the latter too large: Both which conſpire to render her Orbit more curved in the Quadratures than in the Syzygies, ſo that ſhe runs off farther from

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(*n*) See the Lunar Irregularities enumerated in Chap. VIII.

the Earth in the Quadratures, and comes nearer in the Syzygies than ſhe would otherwiſe do, deſcribing an Orbit, one of whoſe Axes, *viz.* that which paſſes through the Quadratures, is longer, than that which paſſes through the Syzygies. *Which is a third Irregularity.*

4. The Gravity of the Moon towards the Earth in the Syzygies, being twice as much diminiſhed by the Action of the Sun, as it is augmented in the Quadratures; if we take a whole Revolution together, it may be conſidered as diminiſhed only. In the Perihelion therefore, at which Time the Earth and Moon are neareſt the Sun, it will be diminiſhed the moſt of all; that Diminution (as ſhewn above) being inverſely as the Cube of the Sun's Diſtance, and ſo the Gravity or Tendency of the Moon to the Earth will be the leaſt. On which Account ſhe will run out into a greater Orbit; and ſo her periodical Time will be greater, than when the Earth is in its Aphelion. *Which is a fourth Irregularity.*

5. When the Moon is in the Quadratures, the Action of the Sun (as ſhewn above) increaſes the Tendency of the Moon to the Earth in Proportion to her Diſtance from thence, this Force ſuperadded to the Action of the Earth upon the Moon, which decreaſes as the Square of the Diſtance increaſes, occaſions that Force to decreaſe as you go from the Earth, *ſlower* than it would otherwiſe do; or which is the ſame Thing to increaſe ſlower as you go the

other way. But by Prop. 9. of the foregoing Chapter, when a retaining Power decreaſes or increaſes ſlower than the Square of the Diſtance increaſes or decreaſes, and the Planet deſcribes an Ellipſe, the *Linea Apſidum* of that Planet will go backwards; the *Linea Apſidum* therefore of the Lunar Orbit, when ſhe is in the Quadratures, goes backwards: When the Moon is in the Syzygies the Action of the Sun diminiſhes her Tendency towards the Earth, and thereby make it decreaſe as you go from, or increaſe as you go to the Earth, too faſt; and ſo the *Linea Apſidum* at that Time goes forwards, But the Diminution in the Syzygies exceeds the Augmentation in the Quadratures, and ſo the *Linea Apſidum* goes farther forwards than backwards every Time, till at length it revolves quite round according to the Order of the Signs. *Which is a fifth Irregularity.*

6. When the Gravity by which a Planet is retained in an Orbit that is excentrical, decreaſes or increaſes *too faſt*, the Planet when going off from the Seat of the retaining Power, that is, towards its upper *Apſis*, will go off too far; and when it is coming to its lower *Apſis*, it will approach too near; and ſo the Excentricity of its Orbit will be increaſed. When its Gravity decreaſes or increaſes *too ſlow*, the Planet will not in the former Caſe go off far enough, nor come ſo near to the Seat of the retaining Power in the latter, as it ought to do; in this Caſe therefore, the Excentricity of its

Orbit will be diminiſhed. But the Tendency of the Moon to the Earth when in the Syzygies, decreaſes or increaſes too faſt, the Excentricity of her Orbit is thereſore at that time the greateſt; and on the contrary it is the leaſt when ſhe is in the Quadratures. And if we compare ſeveral Revolutions of the Moon together, we ſhall find, then when the *Linea Apſidum* is in the Quadratures, the Excentricity of the lunar Orbit will be the leaſt of all; becauſe in that Situation of the *Linea Apſidum*, the Difference between the Tendency the Moon has to the Earth in one of the Apſes, and that which it has in the oppoſite one, is the leaſt of all: Whereas when the *Linea Apſidum* is in the Syzygies, that Difference will be the greateſt; and thereſore the lunar Excentricity will be ſo too. *Which is a ſixth Irregularity.*

7. We have hitherto been conſidering ſuch Irregularities in the Courſe of the Moon as would happen if its Orbit were coincident with the Plane of the Ecliptic: But as it is not ſo, there will ariſe others; in order to account for which, it will be proper to premiſe the following Conſiderations.

Firſt, That when the Line of the Nodes is in the Syzygies, the Plane of the Moon's Orbit paſſes through the Center of the Sun as well as through that of the Earth, and ſo the Moon in that Situation of the Nodes, is not drawn

Fig. 36.

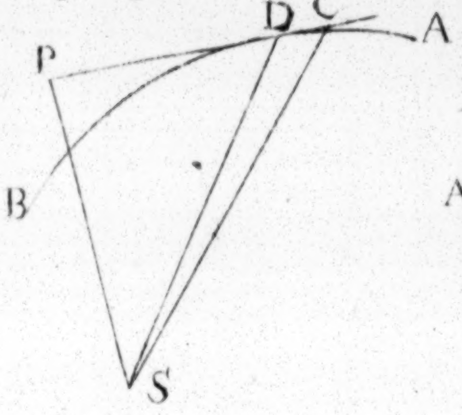


Fig. 37.

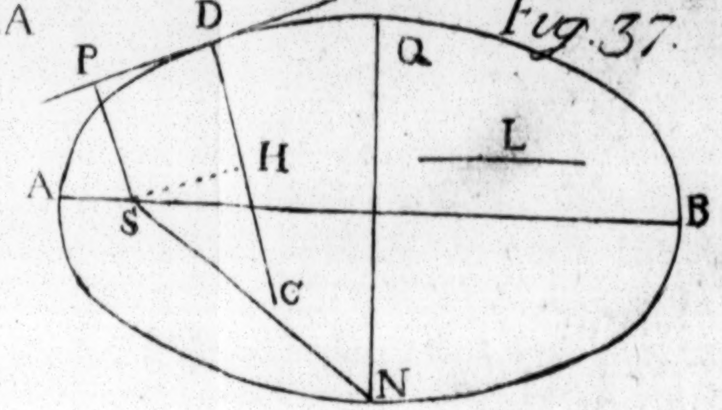


Fig. 39.

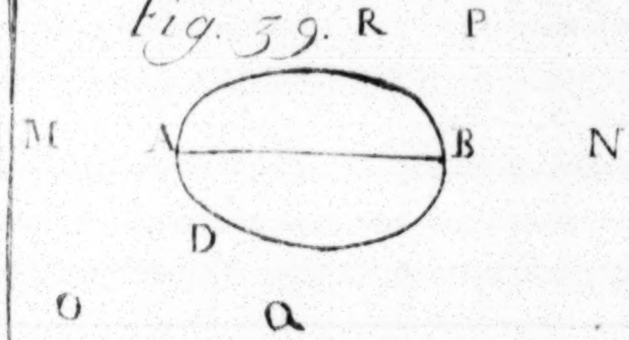


Fig. 38.

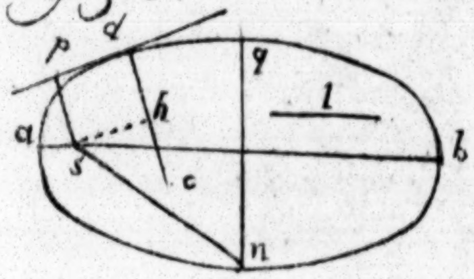


Fig. 41.

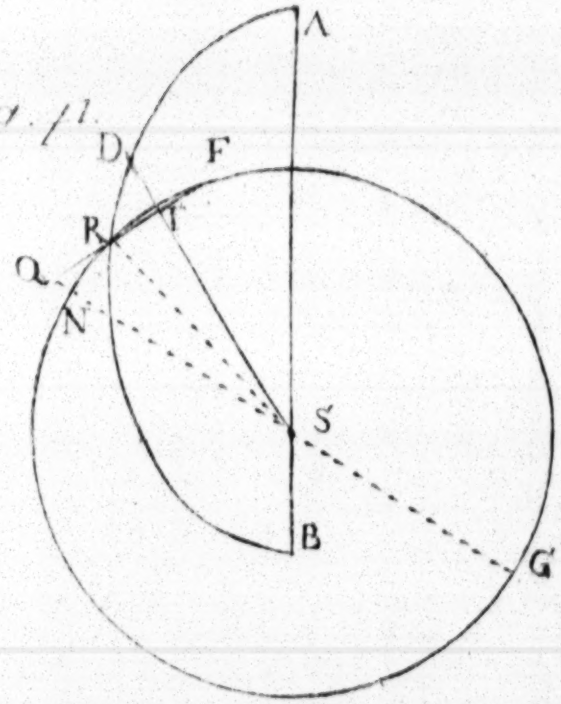
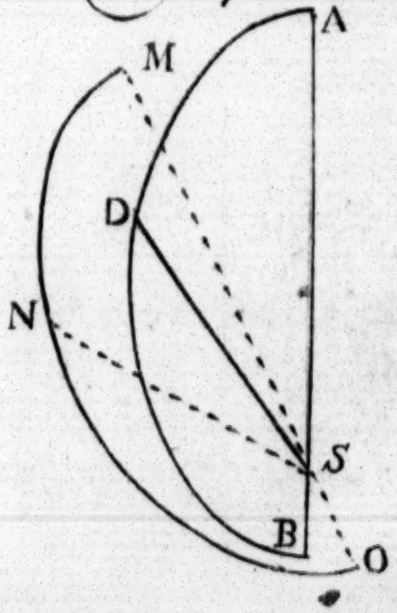


Fig. 40.



drawn out of the Plane of her Orbit by the Sun.

Secondly, That when the Line of the Nodes is in any other Situation, and the Moon not in one of the Nodes, ſhe is continually drawn out of the Plane of her own Orbit, on that Side on which the Sun lies. For inſtance, if the Plane of her Orbit produced paſſes above the Sun, the Sun draws her downwards; if on the contrary, the Plane of her Orbit produced paſſes below the Sun, it draws her upwards.

From theſe two Conſiderations it follows, that when the Line of the Nodes is not in the Syzygies, and the Moon having paſſed either of the Nodes, has got out of the Plane of the Ecliptic on either Side, the Action of the Sun occaſions the Moon to return back to the Plane of the Ecliptic ſooner than ſhe otherwiſe would do; but where the Moon enters that Plane, there is the next Node; ſo that each Node does as it were come towards the Moon, meeting her part of the Way. And the nearer the Line of the Nodes is to the Quadratures, the greater is this Effect, becauſe in that Caſe the Sun is the fartheſt of all from the Plane of the lunar Orbit produced. So that the Line of the Nodes goes backwards the faſteſt of all, when it is in the Quadratures; and not at all in the Syzygies. *Which is the ſeventh Irregularity.*

8. Again, when the Nodes are in the Quadratures, and the Moon has lately paſſed one

of them, and is approaching that Syzygy that is next the Sun, the Action of the Sun upon the Moon prevents her aſcending ſo far; that is, departing ſo far from the Ecliptic as ſhe otherwiſe would do; and ſo diminiſhes the Inclination of her Orbit to the Ecliptic: And as ſhe goes on to the next Quadrature, by haſtening her Deſcent thither, it occaſions the Moon to croſs it in a larger Angle than ſhe would otherwiſe do; and ſo increaſes the Inclination of the Orbit as much as it diminiſhed it before. And, for the ſame Reaſon, while the Moon paſſes from that Quadrature to the oppoſite Syzygy, the Action of the Sun decreaſes the Inclination of her Orbit, and increaſes it again in her Paſſage from thence to the next Quadrature. All which needs no farther Illuſtration, unleſs the following Inſtance may be of Uſe. If you toſs a Stone up into the Air, the Action of the Earth upon the Stone prevents it from riſing ſo high as it would otherwiſe do; and if a Stone be thrown down obliquely, the ſame Action by bending its Courſe towards the Earth all the Way, makes it ſtrike the Earth in a larger Angle than it would otherwiſe do.

When the Nodes are in the Syzygies, the Inclination of the lunar Orbit to the Plane of the Ecliptic is neither increaſed nor diminiſhed: The Sun being then in the Plane of the Moon's Orbit produced.

But while the Nodes are paſſing from the Syzygies to the Quadratures, the Inclination of the Moon's Orbit to the Plane of the Ecliptic, is diminiſhed in every Revolution of the Moon; and while they are paſſing from thence to the Syzygies, it is continually increaſing. So that the Inclination of the lunar Orbit is the greateſt of all when the Nodes are in the Syzygies, and leaſt when they are in the Quadratures. *Which is an eighth Irregularity.* But this requires a particular Explanation.

Let then S in Fig. 46. repreſent the Sun, NFDG the Plane of the Ecliptic, QR the Quadratures, and MO the Syzygies. And let NLDI be the Orbit of the Moon; and ſuppoſe the Nodes at N and D in the middle between the Syzygies and the Quadratures. Farther, let there be a Point H in the Ecliptic oppoſite to the Point S, and let the Orbit NLDI be ſo inclined to the Plane of the Ecliptic, that if it were extended every Way, it would paſs above S and below H. Then becauſe when the Moon is nearer the Sun than the Earth is, ſhe is attracted towards the Sun more than the Earth is; and when ſhe is farther off, the Earth is attracted more than ſhe is, in which caſe ſhe is therefore as it were attracted the other way; let us imagine a Sun at S, and another at H; and let it be remembered that the Orbit of the Moon produced, paſſes above S and below H: And let the Moon be paſſing from N towards L. Then the Attraction here being towards S, and the Orbit produced being above S, it is obvious that the Moon will not paſs to L but to A, a Point between L and F, deſcribing the Curve NA; ſo that the Inclination of the lunar Orbit is perpetually diminiſhed, while the Moon is paſſing over 90 Degrees from the Node N, it being a quarter of a Circle from N to A. To avoid Confuſion in the Figure, let us ſuppoſe that the Moon came to L. In going from thence to the next Quadrature at B, which is 45 Degrees, the Attraction of S prevails ſtill, becauſe the Moon is as yet nearer to S than the Earth is; and therefore as the Orbit produced is above S, and the Moon going downwards, the Attraction of S haſtens her Deſcent, and ſo makes her deſcribe the Curve LC inſtead of LB, which if produced would make with the Plane of the Ecliptic, a larger Angle than her Orbit LB does at D; in going over this 45 Degrees therefore the Inclination of her Orbit is increaſed. Let us now ſuppoſe her going from B towards D; the Attraction here lies towards H, becauſe ſhe is now paſt the Quadrature; and ſhe is tending to a Point below H; H therefore attracts her upwards, making her deſcribe the Curve BE, inſtead of BD, which is about 45 Degrees more, and makes a leſs Angle with the Plane of the Ecliptic, than BD does: Not that E falls beyond D, it only happens ſo in the Figure, becauſe we ſuppoſed the Moon to move from L and B inſtead of A and C. Upon the whole therefore, while ſhe moves from N to D, the Inclination of her Orbit is diminiſhed during three Parts out of four of her Paſſage. In like manner it is diminiſhed by the Attraction of H, while ſhe goes from D to I, and augmented by the ſame Attraction in going from thence to K, and diminiſhed again between K and N. Add to this, that while the Moon moves from N to L, or from D to I, the diſturbing Force, whether of S or H, is much more conſiderable than it is when ſhe is between L and D, or I and N; becauſe in the former Caſe, the Difference between her Diſtance from the Sun, and that of the Earth from the ſame, is greater than it is in the latter. While the Nodes therefore are between M and R, and O and Q; that is, while they are paſſing from the Syzygies to the Quadratures, the Inclination of the lunar Orbit is continually diminiſhing.

For though we have ſuppoſed the Nodes equally diſtant from the Quadratures and Syzygies, the like Effects will happen, though different in Degree, when they are nearer to the one than to the other; as is eaſy to imagine.

Let now the Nodes be in the mid Way betwen R and O, and Q and M, as in Fig. 47. and let the lunar Orbit produced paſs above S and below H as before, and let the Moon be coming from the Node D towards B. She being there in the Power of H, and moving in a Plane which if produced paſſes below H, will be attracted upwards thereby, ſo as to deſcribe the Line DC inſtead of DB; by which means the Inclination of her Orbit will be increaſed. In going from thence to I, ſhe is in the Power of S, which lies below her Orbit produced, and ſo her Aſcent will be diminiſhed, and ſhe will go to A, inſtead of I; whereby the Inclination of her Orbit will be leſſened; and afterwards as ſhe goes towards N ſhe will be attracted downwards all the Way coming to E inſtead of N; by which means the Inclination of her Orbit is again increaſed. So that upon the whole, it is increaſed three Parts out of four of her Paſſage from Node to Node; for the like will hold in her Paſſage through the other Part of her Orbit, and as well when the Nodes are not in the middle between the Quadratures and the Syzygies, as when they are, except in Point of Degree. And for the Reaſon mentioned in the other Caſe, the Force which augments the Inclination of the Orbit, is ſuperior for the Time being, to that which diminiſhes it. While the Nodes therefore are paſſing from the Quadratures to the Syzygies, the Inclination of the Moon's Orbit to the Plane of the Ecliptic is continually increaſing. Which is what remained to be made out.

All theſe Irregularities are greater when the Earth is in its Perihelion, than when it is in its Aphelion, becauſe as was obſerved above, the Effect of the Sun's Action whereby they are produced, is inverſely as the Cube of its Diſtance from the Earth. They are alſo greater when the Moon is in Conjunction with the

Sun, than in Oppoſition for the ſame Reaſon; for the Earth and Moon taken together, are nearer the Sun in the former Situation of the Moon, than they are in the latter.

II. Let us now imagine that the Circle NLDI is a ſolid Ring like that of *Saturn*, and that it moves round its Center T the ſame Way the Moon does round the Earth; it is obvious that every Point of this Ring, will endeavour to put on the ſame Motion that we have ſhewn the Moon to do: That is, that every Point in its Paſſage from N to L, will endeavour to move in the Line NA, (ſee Fig. 46.) every Point between L and B, will endeavour to deſcribe the Curve LC, and every one between B and D, the Curve BE; and the like for Fig. 47. So that the Ring as to the Motion of its Nodes, and its Inclination to the Plane in which its Center moves, will be affected in the ſame manner that the Orbit of the Moon is; and therefore its Nodes, when in the Syzygies, will ſtand ſtill, and its Inclination to the Plane of the Ecliptic will be the greateſt: In all other Situations the Nodes will go backwards, and faſteſt of all when in the Quadratures, at which Time the Inclination of the Ring will be the leaſt.

Let us now ſuppoſe that there is a Redundancy of Matter ſurrounding the Earth in the equatoreal Parts thereof; or in other Words, that the Earth is an oblate Spheroid, having its equatoreal Diameters longer than that which paſſes through the Poles, as we ſhall ſhew it to be in the next Chapter. This Redundancy of Matter, will, like a Ring ſurrounding the Earth and fixed to it, endeavour to put on the abovementioned Motions, and thereby communicate them to the Earth itſelf. The equinoctial Points therefore which anſwer to the Nodes of the Ring, when they are in the Syzygies, that is, at the Equinoxes, will ſtand ſtill, and the Inclination of the Equator to the Plane of the Ecliptic will be the greateſt; in all other Situations they will go backwards, and faſteſt of all at the Summer and Winter Solſtices, at which Times they are as it were in the Quadratures with the Sun; and then the Inclination of the abovementioned Plane will be the leaſt.

III. From hence it follows, that the Axis of the Earth, being perpendicular to the Plane of the Equator, changes therewith its Inclination to the Plane of the Ecliptic twice in every Revolution of the Earth about the Sun. For inſtance, it increaſes while the Earth is moving from the Solſtitial to the Equinoctial, and diminiſhes as much in its Paſſage from the Equinoctial to the Solſtitial Points. Which Phænomenon is called the Nutation of the Poles.

IV. Another Phænomenon and of the ſame kind with the lunar Irregularities, is the ebbing and flowing of the Sea; only, as thoſe ariſe from

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Fig. 42.

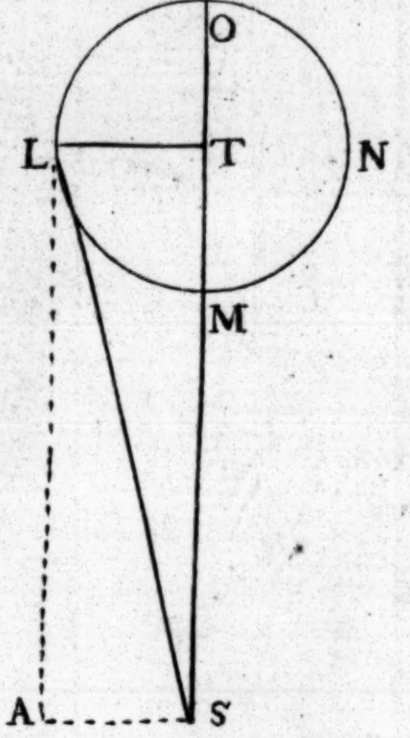


Fig. 43.

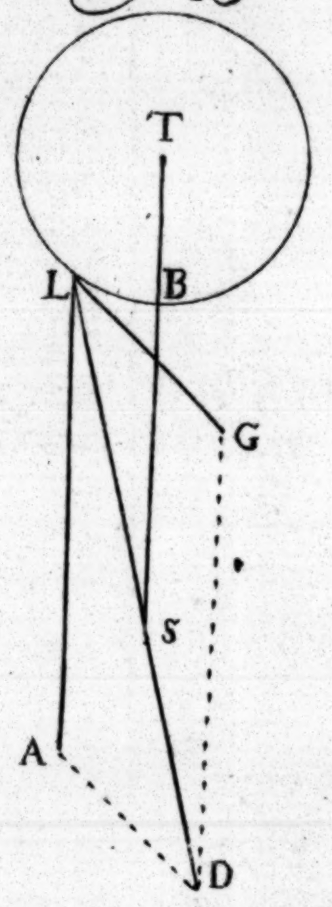


Fig. 45.

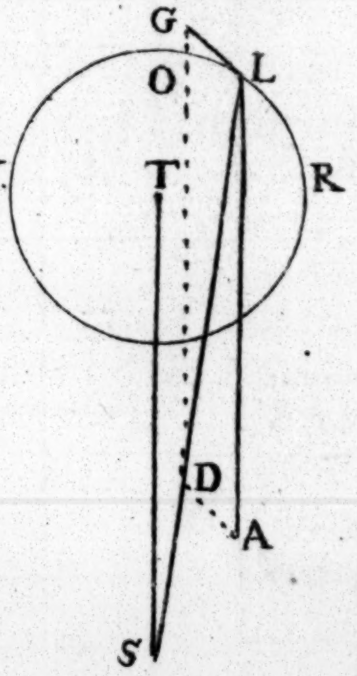
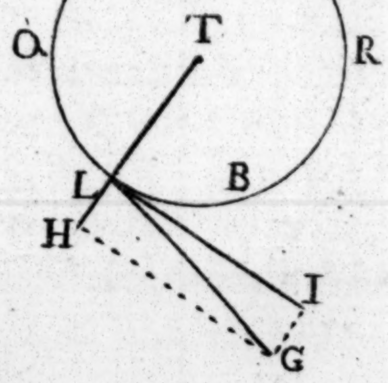


Fig. 44.



from the Action of the Sun upon the Moon, this is owing to the Influence both of the Sun and Moon upon the Waters of the Ocean; and is to be accounted for upon the ſame Principles, after the following Manner.

Let the Point L in Fig. 48. repreſent the Moon, MNOP, the Earth, whoſe Center is C. And let G be the common Center of Gravity of the Moon and the Earth. Which latter let us ſuppoſe ſurrounded with Water to a great Depth. Then, according to what was obſerved at the End of the laſt Chapter, theſe two Bodies continually revolve about the Point G, the Point M deſcribing the Circle AB; the Point C, the Circle NP; and the Point O, the Circle EF; and all in the ſame periodical Time; conſequently by Propoſition the third of the foregoing Chapter, the Forces they require to retain them in thoſe Circles, ought to be to each other as their Diſtances from the Point G: That is, as GM, GC and GO. Conſequently the Point O, which for Diſtinction ſake we will call the *Nadir*, requires a greater Force than the Center C; and the Center, a greater Force than the Point M, which we will call the *Zenith*. Now theſe Points are retained in thoſe Circles by the Moon at L, conſequently the *Nadir* which requires the moſt, is attracted the leaſt, as being fartheſt off; and the *Zenith* which requires the leaſt, being the neareſt, is attracted the moſt; that is, the *Nadir* is attracted too little, and the *Zenith* too much:

The obvious Conſequence of which is, that the Water both in the *Zenith* and *Nadir*, will endeavour to leave the Center C; or, in other Words, will loſe part of its Weight. But the Water at N and P will have its Weight augmented, juſt as the Tendency of the Moon at L (in Fig. 42.) towards T was ſhewn to be augmented by the Action of the Sun at S. So that the Water at N and P will be heavier than an equal Quantity at M or O. And conſequently the Surface of the Waters at N and P will ſubſide, and that at M and O will riſe, 'till the Equilibrium be reſtored. On which Account, the Form of the Earth, or rather the Sea, will become an oblong Spheroid or Oval, as repreſented by NKMH, in Fig. 49. whoſe longer Axis produced paſſes through the Moon at L. As therefore the Moon turns round the Earth once a Day, this Oval of Waters turns with her, occaſioning thereby the two Floods and Ebbs obſervable in each 25 Hours: or to ſpeak more accurately, the Oval of Waters keeps pace with the Moon in her Monthly Courſe, while the Earth in the mean Time, by its Rotation about its Axis, carries each part of its Surface from Ebb to Flood, and from Flood to Ebb continually.

And as the Moon thus raiſes and depreſſes the Water, the Sun does the ſame; but in a much leſs Degree, on Account of the ſmall Proportion the Semidiameter of the Earth bears to the vaſt Diſtance of the Sun; for, as was ſhewn of the Moon, the Force of the Sun whereby it diſturbs her Motion, was proportionable to the Relation the Diſtance of the Moon from the Earth bears to that of the Sun from the ſame, which in the Caſe before us, is the Relation the Semidiameter of the Earth bears to the Diſtance of the Sun, which Relation is very ſmall.

When the Moon is in Conjunction or Oppoſition with the Sun, the Tides which each of them endeavours to raiſe are in the ſame Place, which is the Reaſon they are ſo large at thoſe Times. Whereas when the Moon is in the firſt or laſt Quarter, the Sun being in the Meridian when the Moon is in the Horizon, depreſſes the Water where the Moon raiſes it, on which Account, the Tides are then (*cæteris paribus*) the leaſt of all.

On the full and new Moons, which happen about the Equinoxes, at which Time the Luminaries are both in the Equator or near it, the Tides are the greateſt of all, on the three following Accounts; in the firſt Place, the two Eminences of Water are at the greateſt Diſtance from the Poles, and ſo the Difference between Ebb and Flood is more ſenſible; for if thoſe Eminences were at the Poles, its obvious we ſhould not perceive any Tide at all: Secondly, the equatoreal Diameter of the Earth produced paſſes through the Moon, which Diameter is longer than others, and ſo there is a greater Diſproportion between the Diſtances of the

Zenith,

Zenith, Center and Nadir, from the Center of Gravity of the Earth and Moon, than at other Times: Thirdly, the Water riſing higher in the open Seas, it ruſhes to the Shores with greater Force, where being ſtopped, it riſes higher ſtill; for it not only riſes at the Shores in Proportion to the Height it riſes to in the open Seas, but alſo according to the Velocity it flows with from thence againſt the Shore. The Reaſon why the Spring Tides, which happen a little before the vernal and after the autumnal Equinox, *viz.* in *February* and *October* are the greateſt, is becauſe the Sun is nearer the Earth in the Winter than in the Summer, and ſo the Tides, which otherwiſe would be the greateſt at the Equinoxes, are ſo, a little before the former, and as much after the latter.

We have hitherto conſidered the Tides in general; we muſt now ſee what happens as to Places of different Latitudes. All which will be eaſily underſtood by the help of the 49th Figure: In which let AFD repreſent the Earth whoſe Center is T, the Poles P and O, this the South, the other the North Pole, EQ the Equator, and the Circles FH and KD two Parallels of it, the laſt on the North Side of it, and the firſt on the South Side of it. Let the Fluid ſurrounding the Earth, form itſelf into an oblong Spheroid, whoſe longer Axis HK produced, paſſes through the Moon at L. The right Lines TK or TH, reckoning from the

Center, will repreſent the greateſt Height of the Water, and ſuppoſing NM perpendicular to KH, TN or TM will denote the leaſt, and will repreſent the Height of the Water in all Parts of the Globe through which that Circle NM paſſes. The right Lines TE, TF, TQ and TD, ſuppoſing them drawn, will ſhew the Height of the Water in the reſpective Places E, F, Q and D. Let us now conſider a Place, which by the diurnal Motion of the Earth, deſcribes the Parallel KD: When this Place is at K, the Height of the Water TK, is the greateſt; that is, it is high Tide or Flood when the Moon L is in the Meridian; but afterwards in the ſame Place, the Height of the Water is the leaſt, when the Place is come to X, and again it is Flood when the Place is come to D. But becauſe TK is greater than TD, in the preſent Caſe when the Moon is on the North Side the Equator, the Height of the Sea will be greater, when the Moon is in that Part of the Meridian which is above the Horizon, than when it is in that which is below it. Likewiſe TH is greater than TF, and therefore in a Place which lies under the Parallel FH, or on the other Side the *Line*, the greateſt Height of Water that happens when the Moon is on that Side, is alſo when ſhe is in that Part of the Meridian that is above the Horizon of that Place. Hence it is, that the Moon in the Northern Signs makes the greateſt Tides on our Side the *Line*, when ſhe is above the Earth;

and in the Southern Signs when ſhe is below it. But on Account of that libratory Motion of the Waters, by which they thus riſe and fall alternately, and which would continue ſome Time, although the Sun and Moon ſhould ceaſe to act, the Difference between thoſe Tides which happen when the Moon is above and below the Horizon, is not ſo great as it would otherwiſe be; and the higheſt Spring Tides are not exactly in the new and full Moons, but happen generally three or four Tides after them, and ſometimes later: Becauſe when the Luminaries come to act more forcibly than ordinary, as being in Conjunction or Oppoſition, the Waters will librate backwards and forwards ſeveral Times, before they arrive at their greateſt Height.

Things would be thus, if the Globe of the Earth were covered with Water of a ſufficient Depth, but the Continents which ſtop the Tide; the Straights between them and the Iſlands, and the Shoalneſs of the Sea in ſome Places, which are Impediments to the free Courſe of the Water, cauſe many Exceptions to what has been laid down; and in particular, that even in the open Ocean the Time of high Water is not, when the Moon comes to the Meridian, but always ſome Hours after it. But to be particular in ſuch Circumſtances is not the Deſign of this Treatiſe. See more in *Philoſoph. Tranſact.* No. 226. However, there is one Thing, which becauſe I don't find it taken

Notice of by others, I ſhall juſt mention; and that is, that at both the Summer and Winter Solſtices, there ought to be but one Tide in 24 Hours within the arctic and antarctic Circles; to ſhew this, let KD be the Tropic of Cancer, and let the Sun and Moon be perpendicular over a Point of that Tropic, as at L: Then as obſerved before, there will be an Ebb at N, but becauſe NPK is a quarter of a Circle, N will be a Point of the arctic Circle, and the high Water under that Circle will be at A, the Point oppoſite to N; ſo there will be but one Flood and one Ebb in 24 Hours Time: And as is obvious enough, the ſame will happen in all Places within that Circle, except at the Pole itſelf, where there is no Tide at all. The ſame Things will alſo happen at the other Solſtice, when the Luminaries are in the Tropic of Capricorn. And in thoſe Parallels which lie between the abovementioned Circles and the Equator, each Ebb will be nearer the Time of the leſſer Flood, than it will be to that of the greater. Thus, in the Parallel KD, the Ebbs will be at X, and in the Parallel FH at Y, which Points are nearer to F and D the leſſer Heights of the Water, than to H and K the greater ones. The Reaſon it does not happen exactly in this Manner, at leaſt not in Point of Degree, is that Riſing and Falling or Libration of Waters mentioned above, by which the Ebbs and Floods of each Place are rendered leſs unequal than they ought, according to the Theory, to be.

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## CHAP. XX.

## *Of the* Figures *of the* Heavenly Bodies.

WHEREAS the Heavenly Bodies do not conſiſt wholly of ſolid Matter, but are in all probability partly fluid and partly ſolid, like our Earth, or at leaſt were ſo at firſt; thoſe which have no Motion about their Axes, if ſuch there be, will, from the mutual Gravity and Attraction of their Parts among themſelves, ſettle into a ſpherical Form. But as to ſuch as revolve about their Axes, all their Parts will endeavour to receed from the Axis of their Motion, and thereby the equatoreal Parts where the Motion is the quickeſt, will tend leſs towards the Center than the reſt; their Endeavour to fly off from the Axis about which they revolve, taking off part of their Tendency that Way; ſo that thoſe Parts will become lighter than ſuch are nearer the Poles. The polar Parts thereſore will preſs in towards the Center, and raiſe the equatoreal Parts, till the Quantity of Matter in the latter is ſo far increaſed, as to compenſate for its Lightneſs, and an Equilibrium be reſtored. On which Account, the Form

PART IV. PLATE XII. *Pag.* 274.

*Fig.* 46.

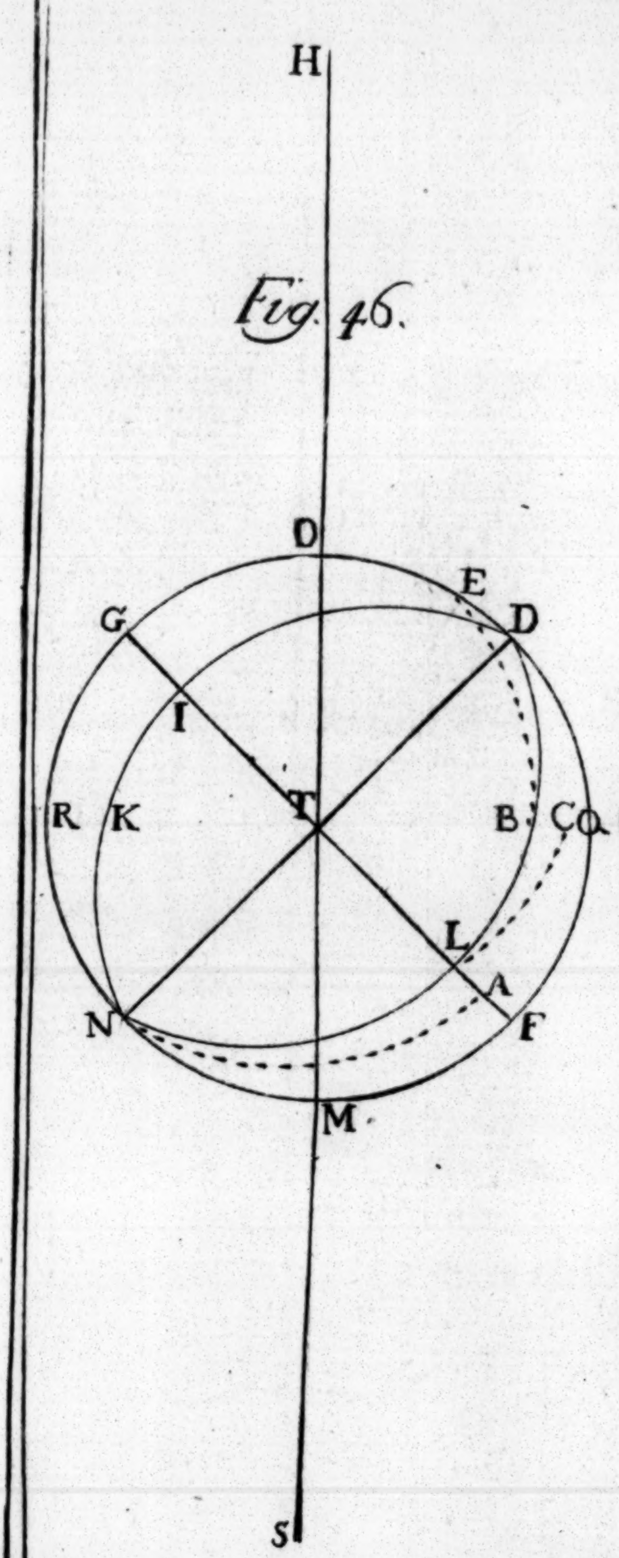
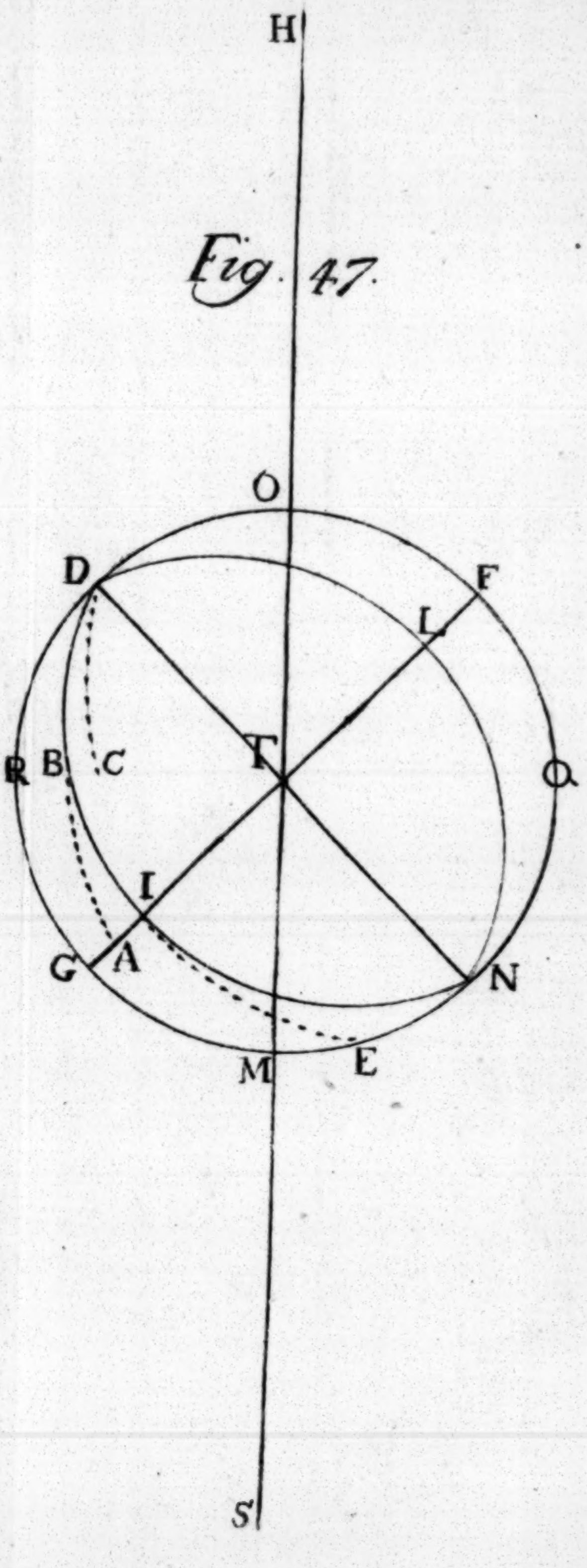


*Fig.* 47.



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Form they will aſſume, will be that of an oblate Spheroid, whoſe ſhorter Axis paſſes through the Poles. And other Circumſtances remaining the ſame, the faſter the Bodies revolve, the more oblate or flat will their Form be; accordingly the Axis of the Planet *Jupiter*, which Planet turns round its Axis in leſs than 10 Hours, is, as appears from the Obſervations of Mr. *Flamſtead* and Monſ. *Caſſini*, no greater with reſpect to the Diameter of its Equator, than in the Proportion of ſeven to eight.

By Virtue only of the Rotation of the Earth about its Axis, the Weight of Bodies at the Equator is leſs than at the Poles, in the Proportion of 288 to 289; that is, a Body which at the Poles would weigh 289 Pounds, would, if carried to the Equator, loſe by Virtue of the centrifugal Force alone, one Pound of its Weight, and ſo weigh but 288 Pounds. From hence ariſes, as obſerved above, a ſpheroidical Form of the Earth, and from that ſpheroidical Form ariſes another Diminution of Gravity at the Equator, by which, if the Earth were homogenious throughout, Bodies at the Equator would loſe one Pound in 1121, and ſo on both Accounts taken together, the Gravity of Bodies at the Poles would be to the ſame at the Equator as 230 to 229 (*n*). From whence, if we ſuppoſe

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(*n*) To calculate this, let us ſuppoſe the Semidiameter of the Earth to be 19695539 *Paris* Feet; which on Account of the vaſt Diſtance on the Surface of the Earth *Caſſini* took in meaſuring it, Sir *Iſaac Newton* makes uſe of. Since then the Earth turns round its Axis in 23 Hours 56 Minutes and 4 Seconds, a Body at the Equator moves through 1436,223 Feet in a Second: But the centrifugal Force of a Body revolving in a Circle is equal to the centripetal Force, which would be requiſite to retain it in that Circle, and the Space a Body would fall through in a Second, by Virtue of that centripetal Force, is by Chap. XVIII. Lemma 1. equal to the Square of the Arch deſcribed in a Second, divided by the Diameter of the Circle, that is, in the preſent Caſe to ,0523 Feet, or 7,54064 Lines. Now the Space a Body falls through at *Paris*, by Virtue of the Gravity there, is 15 Feet 1 Inch and $2\frac{1}{18}$ Lines, or 2174,055 Lines. Therefore the centrifugal Force at the Equator is to the Gravity at *Paris* as 7,54064 to 2174,055. But Bodies do not fall at *Paris*, by the whole Force of Gravity, or as they would do at the Poles of the Earth, becauſe they are in ſome Meaſure prevented by the centrifugal Force there. We muſt therefore compute how much that centrifugal Force is, and add it to their Gravity at *Paris*, to find what it is at the Poles. Which may thus be performed.

Let EPQ in Fig. 50. repreſent the Earth, PP its Poles, EQ its equatoreal Diameter. Let A repreſent any Point between the Equator and the Poles, and parallel to the equatoreal Diameter EQ draw the Line AH, then will the centrifugal Force at E be to that at A as the Radius EC, or which is very nearly the ſame, AC, to AH, Chap. XVIII. Prop. 3. for AH is the Radius of the Circle the Point A deſcribes, while the Earth revolves about the Axis PP. But a Body at A is not thrown off by Virtue of this centrifugal Force directly from the Center of the Earth, but from H along the Line HA produced. Let then AB repreſent that Force, which becauſe it does not tend directly from the Center, let it be reſolved into two others, *viz.* BN and AN; the former perpendicular to the Radius, the latter coincident therewith: It is by this latter Force only that the Gravity of a Body at A is diminiſhed. But the centrifugal Force at the Equator was ſhewn to be to that other Force along the Line AB, as AC to AH, and that other is to the Force along AN as AB to AN, which becauſe of the ſimilar Triangles ABN and ACH is alſo as AC to AH, therefore the centripetal Force at the Equator is to that at A, ſo far as it diminiſhes the Gravity of Bodies there, as $AC^q$ to $AH^q$; that is, becauſe AH is the Coſine of Latitude of the Place A, as the Square of the Radius to the Square of the Co-ſine of the Latitude of the Place where the Body is. Now the Latitude of *Paris* is 48 Degrees and 50 Minutes, the centrifugal Force therefore at the Equator is to that

at

ſuppoſe the Gravity of Bodies within the Earth to be directly as their Diſtance from the Center, as it was ſhewn to be in Part I. Chap. 3. § 8. thoſe Numbers will alſo expreſs the Relation between its polar and equatoreal Diameter (*o*). This

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at *Paris*, as the Square of the Radius to the Square of the Coſine of that Latitude, that is, as 7,540 64 Lines to 3,267. Add this to 2174,055 the Gravity by which Bodies deſcend at *Paris*, and we have 2177,322 for the Gravity they fall with at the Poles: Which Number is to 7,540 the centrifugal Force at the Equator, as 289 to 1. So that the Gravity at the Poles is to the Gravity at the Equator, ſo far as it is diminiſhed in this latter Place by the Rotation of the Earth about its Axis, in the Proportion of 289 to 288. But this is not all: For it may be gathered, from what Sir *Iſaac Newton* has demonſtrated in his *Principia*, Book I. Sect. 13. where he treats of the attractive Forces of Bodies not ſpherical, (though by a tedious and intricate Calculation too long to be inſerted here) that ſuppoſing the Earth to be an oblate Spheroid, ſuch as we ſhall determine by and by, and homogeneous throughout, a Body at the Poles even when the Earth is at Reſt, would be heavier than the ſame at the Equator in the Proportion of about 1121 to 1120. The Weight of a Body therefore at the Poles, when the Earth revolves about its Axis, is to the Weight of the ſame at the Equator, in a Proportion compounded of 289 to 288, and 1121 to 1120; that is, in the Proportion of 230 to 229.

*N. B.* A *Line* is the 12th Part of an Inch.

(*o*) To ſhew this, call an equatoreal Column extended from the Surface to the Center, $r$; and a polar Column, $x$; and call the Force of Gravity at the Equator, $p$: then from what was juſt obſerved in the foregoing Note, ſuppoſing the Earth to be at Reſt, the Force of Gravity at the Poles will be $\frac{1121p}{1120}$, becauſe as $1120 : 1121 :: p : \frac{1121p}{1120}$ : and becauſe the Gravity of a Column if it be equally heavy in all Parts, is equal to the Force of Gravity multiplied by the Contents of the Column, the Weight of an equatoreal Column of Matter, if as heavy in all Parts as at the Surface, would, ſuppoſing the Earth to be at Reſt, be equal to $p$ multiplied by $r$, that is, $pr$; and the Gravity of a

This is upon a Suppoſition that the Earth was at firſt fluid, or a Chaos, having its ſolid and fluid Parts confuſedly mixed together; but if we ſuppoſe it at firſt partly fluid and partly dry, as it now is, ſince we find that the Land

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polar Column would for the ſame Reaſon be $\frac{1121px}{1120}$; but whereas if we ſuppoſe the Force of Attraction inverſely as the Squares of the Diſtances, the Gravity of Bodies within the Earth will decreaſe as we go to the Center, Part I. Chap. 3. § 8. where it terminates in Nothing, the Weights of the above-mentioned Columns decreaſing uniformly therewith, will be but half what we made them before, that is, $\frac{pr}{2}$ and $\frac{1121px}{1120\times 2}$. Call the centrifugal Force at the Equator, $n$; then ſince the centrifugal Force decreaſes alſo as we approach the Center, and there terminates in Nothing, the centrifugal Force of a whole Column of Matter will be $\frac{nr}{2}$. Take this away from $\frac{pr}{2}$, the Weight of that Column when the Earth is at Reſt, and the remainder $\frac{pr}{2}-\frac{nr}{2}$ will be the Weight of the ſame when it moves. But to preſerve an Equilibrium of Parts, this Weight muſt be equal to $\frac{1121px}{1120\times 2}$ which was ſhewn to be the Weight of a Column at the Poles.

Which gives us this Equation $\frac{pr}{2}-\frac{nr}{2}=\frac{1121px}{1120\times 2}$

Multiplying by 2, we have $pr-nr=\frac{1121px}{1120}$

Multiplying by 1120, we have $\Big\}$ $1120pr-1120nr=1121px$

Which gives us this Equation $r:x::1121p:1120p-1120n$.

But as was determined in the foregoing Note, $p$ is to $n$ as 289 to 1. Putting therefore thoſe Numbers for $p$ and $n$, in the laſt Step, we ſhall have $\Big\}$ $r:x::230:229$.

That is, the equatoreal Semidiameter is to the polar one, as 230 : 229. Which was to be ſhewn.

is very nearly of the ſame Figure with the Sea, except raiſed a little to prevent its being overflowed, the Earth muſt ſtill be of the ſame Form; for otherwiſe the major Part of the Water would flow towards the Equator, and ſpread itſelf like an Inundation over all the Land in thoſe Parts.

The ſpheroidical Figure of the Earth is greatly confirmed by Obſervations made with Pendulum Clocks, at different Diſtances from the Equator. Firſt of all Monſ. *Richer* in the Year 1672, when at the Iſland *Cayenne*, found that his Clock, which at *Paris* kept true Time, now loſt 2 Minutes and 28 Seconds every Day. Dr. *Halley* going to the Iſland St. *Helena* in the Year 1677, was obliged to ſhorten the Pendulum of his Clock one eighth part of an Inch. With many others, all which compared together make it appear that a Pendulum at the Equator that ſwings Seconds, ought to be about one ſixth Part of an Inch ſhorter than at *Paris*; from whence it appears that the Difference between the Gravity at the Poles and at the Equator, is almoſt twice as great as what ariſes from the Rotation of the Earth about its Axis, and its ſpheroidical Form put together *(p)*. And therefore the Difference between the equatoreal and polar Diameter muſt be ſo too.

From

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*(p)* For the Length of a Pendulum, ſwinging Seconds is proportionable to the Space a Body would fall through in a Second; as being equal to an eighth Part of that Space, Part I. Chap. 6. Prop. 10. And the Space a Body would fall through in a Second, is proportionable to the Force by which it falls; conſequently

From hence it is probable, that the Parts of the Earth which lie near the Center, may be denſer than ſuch as lie nearer the Surface. For on this Suppoſition, and not otherwiſe, is it accountable, that the Gravity at the Equator diſcoverable by Pendulums, and that which ariſes from the above-mentioned Cauſes, ſhould be ſo different (*q*).

So

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quently the Length of a Pendulum ſwinging Seconds under the Equator ought to be to the Length of one that ſhall ſwing Seconds under the Pole, as the Gravities in thoſe Places are to each other: That is, as 229 to 230. The Length of a Pendulum therefore under the Equator, being 438,5 Lines, (as being two Lines ſhorter than one at *Paris*) the Length of one at the Poles ſhould be 440,4 Lines; that is, it ought to be but 1,9 Line longer than the former. Whereas it is found by the Obſervations abovementioned, that the Difference between a Pendulum in the Latitude of *Paris*, and one under the Equator, is as much as that, or rather more; and conſequently the Difference between one at the Equator and one at the Poles would be almoſt twice as much; there being almoſt as much Difference between a Pendulum at the Poles and at *Paris*, as between one at *Paris* and the Equator: And conſequently the Diminution of Gravity is almoſt twice as much as that which ariſes from the Rotation of the Earth about its Axis, and its ſpheroidical Form put together.

(*q*) If the Earth be more denſe at the Center than near the Surface, we may conſider that Redundancy of Matter near the Center over and above what there would be there, was the Earth of uniform Denſity, apart, and as a ſeparate Body from the reſt, then upon Account of the ſpheroidical Form of the Earth, a Body at the Poles is nearer to this redundant Matter than at the Equator, therefore in removing a Body from the Poles to the Equator, its Gravity ſo far as it depends on the Attraction of this redundant Matter, that is, that Part of its Gravity which is owing to that Attraction, is diminiſhed: The whole Gravity of the Body is leſs therefore at the Equator than at the Poles. As therefore the Pendulum diſcovers a greater Difference of Gravity than ariſes from the Rotation of the Earth about its Axis, and the ſpheroidical Form of it conſidered as homogeneous, put together, it is very probable that the Earth is more denſe at the Center than at the Surface.

So that upon the whole, Bodies are lighter at the Equator than at the Poles, on a threefold Account. Firſt, by the centrifugal Force there, in the Proportion of 289 to 288: Secondly, on Account of the ſpheroidical Form of the Earth, or its Flatneſs at the Poles, in the Proportion of 1121 to 1120; and laſtly on Account of the greater Denſity of the Earth at the Center, but in a Proportion not yet ſufficiently determined, for want of knowing the exact Length of a Pendulum that ſwings Seconds at the Poles.

The greater warmth of the Air near the Equator, increaſes the Length of a Pendulum by Rarefaction, on which Account alone, it would be neceſſary to ſhorten it at the Equator; but this, as may be gathered from the Obſervations of *Pitcairn* and *De la Hire*, cannot be between this Latitude and the Equator above one fortieth part of an Inch; for as *De la Hire* obſerved, there was but two third Parts of a Line Difference between an Iron Rod ſix Feet long, expoſed to the Froſt and the Summer's Sun. Beſides, this is allowed for above, in ſaying that a Pendulum ought to be ſhortned but one ſixth part of an Inch.

It follows from hence, that the Tendency of heavy Bodies upon the Surface of the Earth, except at the Equator and the Poles, is not towards the Center of the Earth, but towards a Point between that and the Equator. Thus, let EPQ in Fig. 50. repreſent the Earth, PP its Poles, EQ its equatoreal Diameter. The Tendency of a Body as A, upon the Surface of the Earth, between E and P, will not be towards C, but along the Line AD which croſſes the equatoreal Diameter in a Point nearer the Body than the Point C is (*r*).

Another Conſequence of the ſpheroidical Form of the Earth is, that as you go from the polar Parts to the equatoreal, the Degrees upon the Surface of the Earth grow leſs and leſs; that

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(*r*) To determine the Point D, ſay as EC is to AH; that is, as the Radius is to the Co-ſine of the Angle of the Latitude of the Place A, ſo is the centrifugal Force at E, to a fourth Number, which will expreſs the centrifugal Force at A in the Direction AB. Produce therefore the Line HA to B, till AB be of ſuch Length, that it may bear the ſame Proportion to AC, that the Quantity laſt found has to Gravity upon the Surface of the Earth. Compleat the Parallelogram ABCD, and D will be the Point ſought, and the Tendency of an heavy Body will be along the Line AD. Thus ſuppoſe it required to find the Direction in which heavy Bodies tend towards the Earth in at the Latitude of 51 Degrees and 46 Minutes. Since the centrifugal Force at the Equator, as ſhewn above, bears that Proportion to the Force of Gravity, which 1 does to 289, let us take thoſe Numbers (or which will do as well, any other two that are proportionable to them) the former to expreſs the centrifugal Force of a Body at the Equator, and the latter the Force of Gravity. Then ſay, as the Radius is to the Co-ſine of 51 Degrees 46 Minutes, ſo is 1 to a fourth Number, which will be found ,618. This Quantity is therefore to Gravity as ,618 to 289. Therefore AB or its equal CD is to AC in that Proportion. But the Proportion between CD and AC being known, and the Angle at C the Latitude of the Place, *viz.* 51 Degrees 46 Minutes being given, the Angle CAD will be found about 5 Minutes, which is the Meaſure of the Deviation of the Line of Direction of heavy Bodies in that Latitude, from a Line drawn to the Center of the Earth.

that is, a Degree meaſured upon the Meridian near the Poles, contains more Miles than the ſame meaſured near the Equator. The Reaſon of this is, that, as appears by Inſpection of the Figure, an Arch near the Poles comes nearer to a ſtraight Line; that is, it is leſs curved than one near the Equator; the former is therefore an Arch of a larger Circle than the latter; but the larger the Circle, the larger are the Degrees meaſured upon its Circumference. For in going from P towards E, we are not to imagine ourſelves upon the Circumference of a Circle whoſe Center is C, but to paſs continually from one Arch to another, which Arches are Portions of different Circles, the Centers of which approach, as the Arches themſelves become more curved.

As the Waters of the Earth by Virtue of the Attraction of the Moon, and the Revolution of the Earth about the common Center of Gravity of that and the Moon, were ſhewn in Chapter the laſt, to put on the Form of an oblong Spheroid, whoſe Axis produced paſſes through the Moon; ſo in like manner, if we conſider the Moon as we have now done the Earth, we ſhall find, that as ſhe turns round her Axis in the ſame Time ſhe turns round the Earth, and therefore has nearly the ſame Side always towards the Earth, her Figure is that of an oblong Spheroid, whoſe longer Axis points to the Earth; thoſe Parts which lie next the Earth being attracted too much, and alſo having too ſmall a centrifugal Force; and thoſe on the contrary, which lie on the oppoſite Side of her being attracted too little, and having a centrifugal Force too great; which for the Reaſons given in the above-mentioned Place, will neceſſarily give her that Form.

Her Revolution about her Axis would, as in the Earth, give her a contrary Figure, but it is ſo very ſlow, that it is without any ſenſible Effect.

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## Material ERRATA in Part III and IV.

### PART III.

Page 164 Line 2 for ſuppoſed, read diſpoſed.

### PART IV.

| Page | Line | Inſtead of | Read |
|---|---|---|---|
| 12 | 21 | ADME | ADBE |
| 22 | 23 | neither the Sun nor the Earth | both the Sun and the Earth, or round neither of them |
| 34 | 18 | Fig. 2. | Fig. 3. |
| 34 | 32 | XV. | XVI. |
| 45 | 2 | Side | Side of the Equator |
| 48 | 29 | XIV. | XVI. |
| 95 | 13 | can- | cannot |
| 99 | 26 | (*d*) | (*b*) |
| 114 | 4 | Horizon | Heavens |
| 114 | 22 | Semidiameter | a Semidiameter |
| 119 | 29 | VIII. | VII. |
| 122 | 27 | Refraction | Reflection |
| 140 | 14 | when | from the Place where |
| 141 | 29 | Juvinum | Juviſium. |

PART IV. PLATE XIII. *Pag.*

*Fig. 49.*

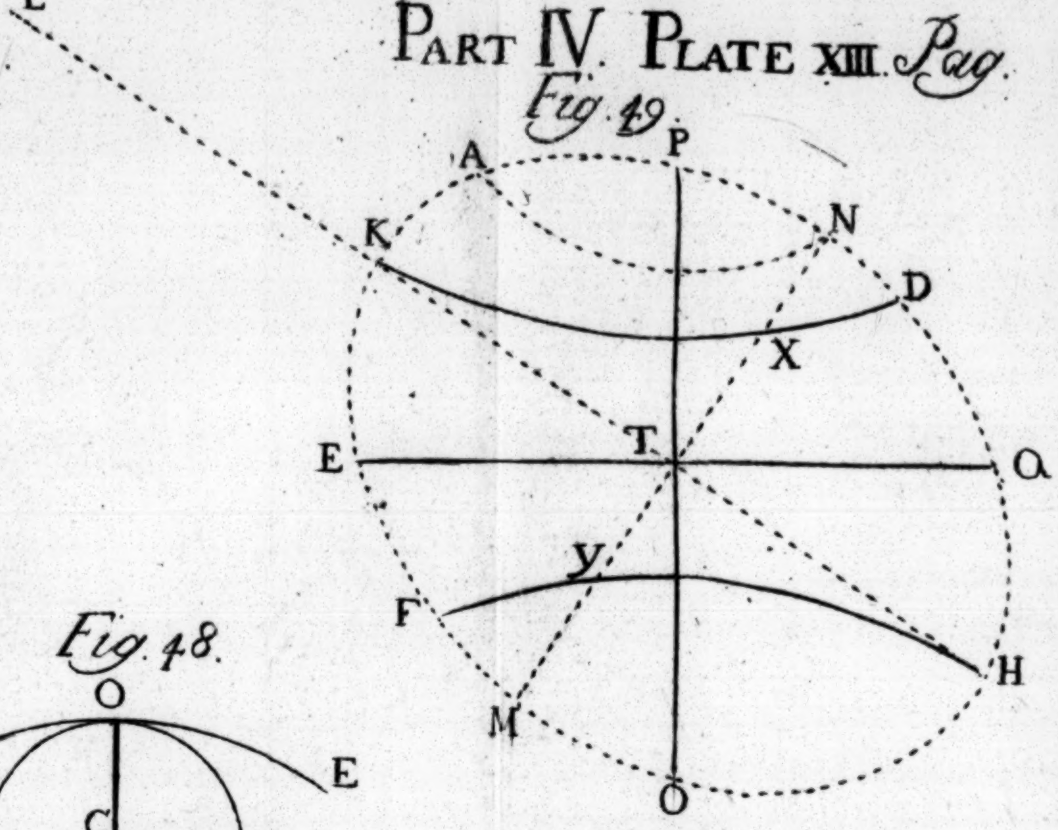


*Fig. 48.*

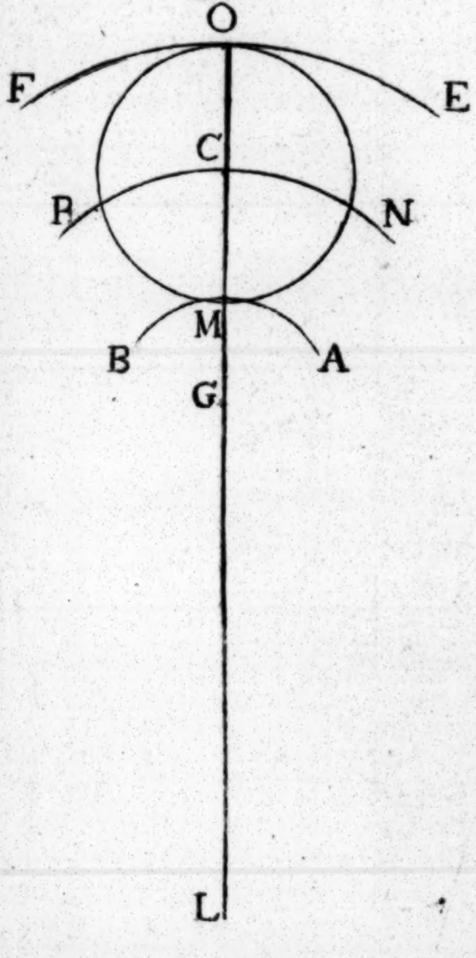
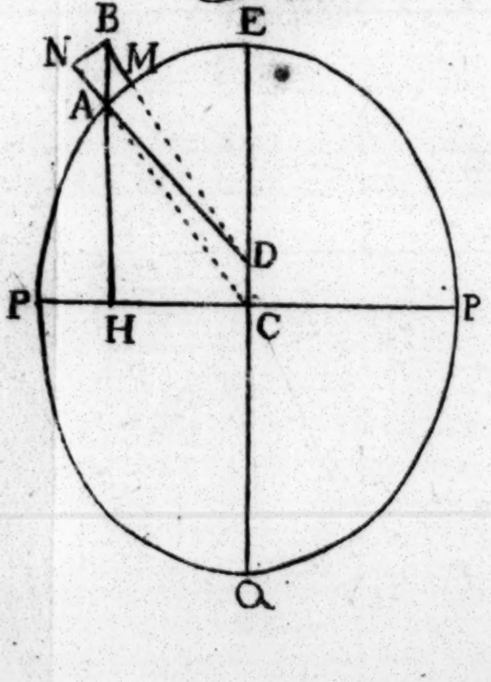


*Fig. 50.*



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